The Conference "Computer Algebra" in Moscow

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The International Conference "Computer Algebra"was held in Moscow, Russia from June 29 till July 2, 2016. The conference web-site is http://www.ccas.ru/ca/conference. Co-organized by the Dorodnicyn Computing Centre (Federal Research Center "Computer Science and Control") of Russian Academy of Sciences and the Peoples' Friendship University of Russia. It was devoted to computer algebra and related topics. The conference was supported by the Russian Foundation for Basic Research under Grant No. 16-01-20379.

1 Invited Talk Abstracts

Convex polyhedron in asymptotic analysis

A.D.Bruno (Keldysh Institute of Applied Mathematics of RAS)

It is shown here that for computation of singular solutions of algebraic and differential equations near their singularities it is convenient to compute some analogues of the Newton polyhedra and to separate the truncated equations by means of the Newton polyhedron. Then for asymptotic expansions of solutions of a certain form we state a theorem that its truncation is a solution of the corresponding truncated equation. We propose here a new form of asymptotic expansions. Moreover computation of convex polyhedron gives global generalization of continued fractions and allows to find the best Diophantine approximations.

On the generalized fast automatic differentiation technique

Y.G.Evtushenko, V.I.Zubov (Dorodnicyn Computing Centre, FRC CSC of RAS; Moscow Institute of Physics and Technology)

The new efficient methodology, designed for the numerical computation of the gradient of functions, is presented. It may be applied to solve a wide range of optimal control problems of complex dynamical systems described by ordinary differential equations and (or) partial differential equations. The essence of this approach is that it displays the canonical formula to calculate the gradient of the cost function with machine precision for the selected approximation of the target functional.

Description of two qubit entanglement space in terms of polynomial invariants: a challenge for computer algebra

V.P.Gerdt (Joint Institute for Nuclear Research)

We consider computational aspects of characterising the entanglement space of two qubit mixed states via the polynomial invariants of local unitary group $SU(2) \times SU(2)$. Although a number of computer algebra based algorithms has been designed for construction of the ring of invariant polynomials, the underlying computations related to two qubits are too hard for those algorithms. In the talk we restrict ourselves to a subset of the two qubit states containing the so-called X-states and investigate its invariant ring.

The features of tensor computer algebra systems

D.S.Kulyabov (Peoples' Friendship University of Russia; Joint Institute for Nuclear Research), *A.V.Korolkova* (Peoples' Friendship University of Russia), and *L.A.Sevastianov* (Peoples' Friendship University of Russia; Joint Institute for Nuclear Research)

Tensorial computer algebra systems are quite different from the scalar ones. Moreover, there are several types of tensor computations. We can formulate the criteria that must be satisfied by a computer algebra system dealing with tensors.

Transforming linear functional systems into fully integrable systems

Z.Li (Key Laboratory of Mathematics Mechanization, Academy of Mathematics and System Sciences, Academia Sinica, China) and *M.Wu* (Shanghai Key Lab of Trustworthy Computing, East China Normal University, China)

A linear (partial) functional system consists of linear partial differential, difference equations or any mixture thereof. In this talk, we present an algorithm that determines whether linear functional systems are ∂-finite, and transforms ∂-finite systems into fully integrable ones. The algorithm avoids using Gröbner bases in Laurent-Ore modules when ∂-finite systems correspond to finite-dimensional Ore modules.

On integration of ordinary differential equations

M.D.Malykh (Peoples' Friendship University of Russia; Moscow State University)

There are several approaches to the description of the concept which is called briefly integration of differential equations in finite terms, or symbolic integration. In the report these approaches and progress in their implementation in CAS are compared.

Computer algebra methods for modeling in celestial mechanics

A.N.Prokopenya (Warsaw University of Life Sciences)

We discuss here the basic computational problems arising when studying various models of celestial mechanics. As an example we consider the planar circular four-body problem formulated on the basis of collinear Euler solutions of the three-body problem. We show that the system under consideration has 18 equilibrium solutions and develop an algorithm for their calculation. Doing relevant symbolic computations, we reduce the Hamiltonian function to the normal form and analyse stability of the equilibrium solutions for different values of the system parameters. All relevant symbolic and numerical calculations are done with the computer algebra system Mathematica.

2 Contributed Talk Abstracts

Liouvillian sequences and convolution

S.A.Abramov (Dorodnicyn Computing Centre, FRC CSC of RAS) and *M.Petkovšek, H.Zakrajšek* (University of Ljubljana)

We demonstrate by examples that a convolution of two hypergeometric sequences need not be Liouvillian, hence neither the ring of d'Alembertian sequences nor the ring of Liouvillian sequences is closed under convolution. On the positive side, we show that d'Alembertian sequences are closed under convolution with rationally d'Alembertian sequences, and Liouvillian sequences are closed under convolution with rationally Liouvillian sequences.

Resonance set of polynomial and its applications

A.B.Batkhin (Keldysh Institute of Applied Mathematics of RAS)

We consider the resonance set of a real polynomial, i.e., the set of all the points of the coefficient space at which the polynomial has commensurable roots. The constructive algorithm of computation of polynomial parametrization of the resonance set is provided. The structure of the resonance set of a polynomial of degree n is described in terms of partitions of the number n. The algorithm for computation of parametric representation of the resonance set is implemented as a library of the CAS Maple. The obtained results are used for solving the problem of formal stability of a stationary point of a multiparametric Hamiltonian problem with three degrees of freedom.

Computing polynomial solutions to hypergeometric systems

D.V.Bogdanov, T.M.Sadykov (Plekhanov Russian University)

We present a Mathematica-based package for computing polynomial solutions to holonomic systems of partial differential equations of hypergeometric type.

Application of computer algebra systems to the study of statistical systems

E.G.Eferina, A.V.Korolkova (Peoples' Friendship University of Russia), *D.S.Kulyabov, L.A.Sevastianov* (Peoples' Friendship University of Russia; Joint Institute for Nuclear Research)

Methodology of stochastization of one-step processes is based on the expansion of the master equation and obtaining the approximate models. It is proposed to treat the master equation with the help of statistic perturbation theory (as in the frame of quantum field theory). For this purpose, the methodology was described and the analytical software complex was constructed to put the main kinetic equation in the operator form in the Fock representation. To solve the resulting equation the software complex generates Feynman diagrams of the corresponding order of perturbation theory. We applied the FORM system as a system of symbolic computation.

A Maillet-Malgrange type theorem for generalized power series

R.R.Gontsov (Institute for Information Transmission Problems of RAS), *I.V.Goryuchkina* (Keldysh Institute of Applied Mathematics of RAS)

There is proposed a Maillet-Malgrange type theorem for a generalized power series (having complex power exponents) formally satisfying an algebraic ordinary differential equation. The theorem describes the growth of the series coefficients.

Calculation of transition probabilities for the atoms and ions with one valence electron in the operational model of quantum measurements using methods of computer algebra

A.V.Gorbachev (Peoples' Friendship University of Russia), *L.A.Sevastianov* (Peoples' Friendship University of Russia; Joint Institute for Nuclear Research)

Recent findings proved the consistency of the Kuryshkin-Wódkiewicz model of quantum measurements with implementation of Weyl-Kuryshkin quantization rule applied to the extended quantum Kepler problem of quantum systems with one valence electron, such as alkali metal atoms. In this work computer algebra methods of Maple were used to formulate a procedure for calculating transition probabilities in quantum systems with one valence electron based on Kuryshkin-Wódkiewicz model of quantum measurements. Calculations of transition probabilities were developed as a module integrated into dedicated QDF package containing mathematical apparatus of the quantum measurement model.

Algorithms for solving boundary value problems for a system of second-order ODE with piecewise constant potentials

A.A.Gusev, O.Chuluunbaatar, S.I.Vinitsky, L.L.Hai (Joint Institute for Nuclear Research), *V.L.Derbov* (Saratov State University)

An algorithm for solving a parametric eigenvalue problem for the second order ordinary differential equation in a finite interval at large values of a parameter is presented. Solutions are sought by matching a numerical

solution in one of the subintervals with an analytic solution in the other subinterval. The efficiency of the algorithm is demonstrated by comparison of calculated solutions with solution of the parametric eigenvalue problem by finite element method on whole interval of independent variable at large values of the parameter.

A symbolic-numeric algorithm for solving the Sturm-Liouville problem at large values of parameter

A.A.Gusev, O.Chuluunbaatar, S.I.Vinitsky, L.L.Hai (Joint Institute for Nuclear Research), *V.L.Derbov* (Saratov State University)

Algorithms for solving the multichannel scattering and eigenvalue problems of the waveguide type for a system of second order ODEs with piecewise constant coefficients in the axis are presented. A comparison of the results obtained by solving the boundary value problem using the finite element method and the method of matching fundamental solutions of the system in the points of discontinuities of potentials is performed.

Application of computer algebra methods for investigation of the aerodynamic forces influence on stationary motions of satellite

S.A.Gutnik (Moscow Institute of Physics and Technology), *V.A.Sarychev* (Keldysh Institute of Applied Mathematics of RAS)

With the help of computer algebra methods the properties of a non-linear algebraic system, which determines the stationary motion of a satellite on a circular orbit by the action of gravitational and aerodynamic moments are investigated. All bifurcation values of parameters at which there is a change of numbers of equilibrium orientations are determined. Evolution of domains in the space of parameters of the system which correspond to various numbers of equilibria are carried out in detail.

About parallelizing recursive block algebraic algorithms: algorithms and experiments

E.A.Ilchenko (Tambov State University)

We describe an algorithm for the decentralized control of parallel computing process which is based on the SPMD computational paradigm and present the results of experiments on a cluster MVS–10P.

Quantum evolution model based on symmetric group

V.V.Kornyak (Joint Institute for Nuclear Research)

We consider a constructive model of quantum evolution based on the natural and standard representations of the symmetric group. Within this model we study some quantum-mechanical problems. Our strategy in the investigation of these problems consists in searching dominant structures and dominant evolutions. For computations we use the systems GAP and Magma, as well as a program written in C.

Realization of Weyl groups on ellipsoids and application to Chevalley-Bruhat ordering

A.V.Loutsiouk (King Mongkut University of Technology Thonbury, Bangkok)

Two geometric realizations of Weyl groups of all finite-dimensional complex semisimple Lie algebras are defined and used for constructing algorithms of Chevalley-Bruhat ordering of the Weyl groups and finding all reduced expressions of all their elements.

MathPartner: computer algebra of new generation

G.I.Malaschonok(Tambov State University)

We give the general characteristics of the mathematical service MathPartner, which is now freely available at mathpar.cloud.unihub.ru. This mathematical service focuses on the most extensive use in science and education, as well as in industry and technology. We discuss the characteristics of the new generation of computer algebra systems and their possible impact on society. We build forecasts of possible cardinal changes that can get the traditional educational system under the influence of new technical capabilities that they provide.

The Laplace transform method in an algorithm of solving differential equations with delayed argument

N.A.Malashonok (Tambov State University)

The method is used for linear differential equations with delayed argument. We construct an algorithm, which is symbolic-numerical. The numerical component concerns representation of functions, involved in the process, by some kind of series.

Calculus from the computer algebra viewpoint

M.D.Malykh (Peoples' Friendship University of Russia; Moscow State University)

The report is about the use of computer algebra systems in education, personal experience of reading mathematical courses at MSU and PFUR and applications of Microsoft Mathematics and Sage.

The analytic complexity of cluster trees in big data analysis

A.I.Normov (Plekhanov Russian University)

We introduce the definition of the analytic complexity of a cluster tree and give an algorithm to compute it.

On determination of satellite unknowns in linear differential systems

A.A.Panferov (Dorodnicyn Computing Centre, FRC CSC of RAS; Moscow State University)

We consider a linear homogeneous differential system with selected unknowns. We present an algorithm to determine a set of satellite unknowns, i.e., unselected unknowns that belong to the same differential extension as the selected ones.

On universal denominator of rational function solutions of partial differential and difference equations

S.V.Paramonov (Dorodnicyn Computing Centre, FRC CSC 0f RAS; Moscow State University)

We consider the problem of finding a universal denominator for linear partial differential and difference equations with polynomial coefficients. In the case of ordinary equations, known algorithms for finding rational function solutions are based on constructing a universal denominator. In the multivariate case a universal denominator does not exist for an arbitrary equation but there are algorithms for difference equations that find certain information about rational function solutions and build a partial universal denominator. We prove that the problem of testing the existence of universal denominators for linear partial differential and difference equations is algorithmically undecidable.

Differential analogue of the primitive element theorem

G.A.Pogudin (Moscow State University)

In 1942 E. Kolchin obtained the following differential analogue of the Primitive Element theorem:

> Let E be a finitely generated differential field extension of a differential field F. Assume that every element of E satisfies an algebraic differential equation over F and F contains a nonconstant element. Then, there exists a in E such that E is generated by a and F.

It was unknown for several decades if the theorem holds in the case when F consists of constants and E contains a nonconstant element. Recently, this question was answered affirmatively.

Construction of resolving sequences for systems given by Ore polynomials

A.A.Ryabenko (Dorodnicyn Computing Centre, FRC CSC of RAS)

A modification of the algorithm (and its implementation in Maple 2015) to construct resolving sequences for the case of linear operator systems given by Ore polynomials is presented.

Derivation of equations for the coefficient functions of the method of cross-sections for the integrated optical waveguide using the Maple symbolic computation package

A.L.Sevastianov (Peoples' Friendship University of Russia), *L.A.Sevastianov* (Peoples' Friendship University of Russia; Joint Institute for Nuclear Research), *A.A.Tiutiunnik* (Peoples' Friendship University of Russia)

We obtain an ordinary differential system for the coefficient functions of the method of cross sections for the guided modes of a smoothly irregular integrated-optical waveguide using a computer algebra system.

Efficient computation of bounds of polynomial roots

D.Ştefănescu (University of Bucharest)

We discuss the efficiency of the computation of bounds for polynomial roots. We overview some classical devices and we discuss recent trends concerning the location of roots of univariate polynomials with real or complex coefficients.

Passare.ru: a web-service for automatic text synthesis in Russian

T.A.Zhukov, T.A.Sadykov (Plekhanov Russian University)

The problems of synthesis of a grammatically correct text in a natural language and of an automated correction of a text are known to be highly complex. They are closely related to the problem of computer-aided translation that can hardly be considered to be solved despite the considerable effort. We present algorithms used in the web-service passare.ru which is aimed at automatic processing of linguistic data.

Gröbner bases of some parametric systems

A.I.Zobnin (National Research University Higher School of Economics), *M.I.Tikhonova* (Moscow State University)

We study Gröbner bases of ideals in polynomial ring $k[x_0, \dots, x_n, \dots]$ in infinitely many variables. These ideals are generated by parametric polynomials of special form. We show that these ideals possess finite equivariant Gröbner basis.