The Second Conference "Computer Algebra" in Moscow

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The conference website: http://www.ccas.ru/ca/conference.

1 Invited Talks Abstracts

On Linear Difference Equations with Factorial Series Coefficients

M. Barkatou (Limoges University)

This talk is based on a joint work Thomas Cluzeau (University of Limoges; XLIM, France) and Carole El Bacha (Lebanese University). It concerns the local study (at infinity) of linear difference systems with factorial series coefficients. After giving some motivations for working with inverse factorial series instead of the usual inverse power series, three main problems will be discussed : P1) Reductio to simple forms, P2) Computing the so called formal regular solutions and P3) Solving systems with first-kind singularity. We will present a new algorithm for computing simple forms. This algorithm allows to determine the nature (regular/irregular) of the singularity at infinity. When the input system has a regular singularity, the reduction to a simple form yields a system of first-kind. An algorithm for solving systems of first-kind will be presented. Computing simple forms is also very useful for solving problem P2) for any kind of systems even in the case of an irregular singularity. A complete algorithm to solve problem P2) will be presented. Finally, we will present our implementations in Maple of our algorithms with a focus on manipulations of factorial series. The talk will be illustrated by examples computed using our implementation.

Calculation of Complicated Asymptotic Expansions of Solutions to ODEs

A.D. Bruno (Keldysh Institute of Applied Mathematics of RAS)

We consider the complicated asymptotic expansions of solutions to a polynomial ordinary differential equation (ODE). They are such series on integral powers of the independent variable, which coefficients are the Laurent series on decreasing powers of the logarithm of the independent variable. We propose an algorithm for writing ODEs for these coefficients. The first coefficient is a solution of a truncated equation. For some initial equations, it is a polynomial. Question: will the following coefficients be polynomials? Here the question is considered for the third (P_3) and sixth (P_6) Painlevé equations. It appears that for

them the second coefficients are polynomials in all cases, but the third coefficient is a polynomial ether always, either under some restriction on parameters, or never.

Computing Solutions of Linear Mahler Equations

F. Chyzak (INRIA, Université Paris-Saclay), *Th. Dreyfus* (Université Claude Bernard Lyon 1), *Ph. Dumas* (INRIA, Université Paris-Saclay), *M. Mezzarobba* (CNRS, Sorbonne Universités, Université Pierre et Marie Curie Paris 6)

Mahler equations relate evaluations of the same function f at iterated bth powers of the variable. They arise in particular in the study of automatic sequences and in the complexity analysis of divide-and-conquer algorithms. Recently, the problem of solving Mahler equations in closed form has occurred in connection with number-theoretic questions. A difficulty in the manipulation of Mahler equations is the exponential blow-up of degrees when applying a Mahler operator to a polynomial. In this work, we present algorithms for solving linear Mahler equations for series, polynomials, and rational functions, and get polynomial-time complexity under a mild assumption. Incidentally, we develop an algorithm for computing the gcrd of a family of linear Mahler operators.

Computer Algebra in Photometric Stereo with Two Light Sources

R. Kozera, A.N. Prokopenya (Warsaw University of Life Sciences)

We discuss here the basic computational problems arising in recovering a shape of the unknown Lambertian surface from two its photometric images taken consecutively under lighting the surface by a pair distant light sources from two different directions. Using the computer algebra system Mathematica, the necessary and sufficient conditions for unique determination of the second-order algebraic surface from its two images are analyzed in a general setting. Theoretical results are illustrated by examples exploiting models of the photometric images of a genuine second-order Lambertian surface. Symbolic computation performed with Mathematica confirms the possibility of a unique reconstruction of such surfaces from their two photometric images and demonstrate that the considered model has more than one solution only in rare cases.

Boolean Functions and Symbolic Computation

M. Petkovšek, L. Vukšič (University of Ljubljana)

We consider symbolic computation in P_2, the algebra of Boolean functions with Mal'cev operations. Questions such as whether a set of functions generates P_2, or if a given function can be generated by a given basis, can be answered using Post's results on the structure of the lattice of subalgebras of P_2. We present a short proof of Post's Functional Completeness Theorem, and some *Mathematica* implementations of the corresponding procedures.

Coalescing Complex Singular Points

L. Teyssier (Strasbourg University)

We study the saddle-node bifurcation in codimension-k families of holomorphic planar vector fields, admitting a persistent invariant central manifold. This situation corresponds generically to the coalescence of $k+1$ simple stationary points into a single multiple (*i.e.* irregular) point. In a previous work with

C. Rousseau, we studied the corresponding family of non-linear Stokes phenomena, measured by a functional modulus encoding completely the conjugacy class of the bifurcation up to local analytic changes of coordinates, parameters and time. We were also interested in the inverse problem, which is far more difficult than for the non-parametric case. This study led us to provide analytic normal forms with simple expressions. Here we investigate to which extent these objects can be computed (numerically or symbolically) and how these computations pertain to the problem of family-wise Liouvillian integrability and to the inverse problem.

Machine Learning Application for the Improvement of the F_4

M.I. Tikhonova (Moscow State University), *A.I. Zobnin* (Higher School of Economics)

Groebner basis computation of a polynomial ideal is an important problem of computer algebra. At the same time it is computationally difficult. For its solution various algorithms were proposed. Most of them use different heuristics. In this paper we examine the applicability of machine learning the optimization of one of the algorithms, namely F_4 algorithm. A range of different experiments were also performed which showed the efficiency of the usage of one of the machine learning methods, known as support vector machine, for the F_4 algorithm.

2 Contributed Talk Abstracts

EG-Eliminations as a Tool for Computing Rational Solutions of Linear q-Difference Systems of Arbitrary Order with Polynomial Coefficients

S.A. Abramov (Dorodnicyn Computing Center, Federal Research Center "Computer Science and Control" of RAS)

We consider systems of linear q-difference equations with polynomial coefficients. The equations of systems can be of arbitrary order. Using EG-eliminations, we propose for such systems, a direct algorithm for finding rational solutions, i.e., solutions whose components are rational functions. The algorithm is direct, i.e., it does not require preliminary cyclic vector method applying, or another type of uncoupling of the system.

Generalized Discriminant of a Real Polynomial

A.B. Batkhin (Keldysh Institute of Applied Mathematics of RAS)

We consider a generalization of a classical discriminant of a real polynomial. This generalization is defined by the linear Hahn operator which decreases the degree of polynomial by one. The structure of the generalized discriminant set, i.e. the set of all values of space of polynomial's coefficients at which the polynomial and its Hahn operator image have equal roots, is investigated. This structure is described with the help of partitions of natural number n – the degree of the polynomial. A constructive algorithm for a polynomial parametrization of the generalized discriminant set in the space of the coefficients of the polynomial is proposed. The basic algorithms described in this paper are implemented as a library for Maple.

Computation of Amoebas of Polynomials in Two Variables

D.V. Bogdanov (Moscow Economic Institute)

The term "amoeba" was firstly introduced in monograph by Gelfand, Kapranov and Zelevinsky (1994), thereafter set of scientific works of various authors was devoted to researching of amoebas. Modern systems of computer algebra allow to solve effectively the wide range of tasks, including "amoebic" theme. The reckoning for universality of such systems is the absence of many built-in functions, including those for computation of amoebas. That's why, we have to develop and program non-trivial algorithms often enough, even in case of typical tasks. This work is focused on MatLab code wrapper, providing convenient web interface for computing and visualization of affine and compactified amoebas of polynomials in two variables. The possibility of Newton polygon visualization both by means of systems of computer algebra and by means of LaTeX graphics is considered. The offered approach realizes the concept of RAD (rapid development of applications) and allows to focus on mathematical issue without routine programming of functions used. The definition and samples of computation of weighted compactification amoebas of polynomial of two variables ($\mathcal{WCA}$) are given. The limiting case is shown, in which the $\mathcal{WCA}$ degenerates into the simplicial complex.

Algorithm for Calculating Interpolation Hermite Polynomials for High-Accuracy Finite Element Method

G. Chuluunbaatar, O. Chuluunbaatar (Joint Institute for Nuclear Research, Dubna), *V. Derbov* (N.G. Chernyshevsky Saratov National Research State University), *V.P. Gerdt* (Joint Institute for Nuclear Research, Dubna), *A. Góźdź* (University of M. Curie-Sklodowska, Lublin), *A.A. Gusev* (Joint Institute for Nuclear Research, Dubna), *P. Krassovitskiy* (Institute of Nuclear Physics, Almaty), *S.I. Vinitsky* (Joint Institute for Nuclear Research, Dubna)

We propose a new algorithm for calculating high-order Hermite interpolation polynomials of the simplex of the Euclidean space implementable in any computer algebra system in analytical form and give their classification and a typical example of triangle element. The basis functions of finite elements referred to as Hermite interpolation polynomials are high-order polynomials, determined from a specially constructed set of values of the polynomials themselves, their partial derivatives, and their derivatives along the directions of the normals to the boundaries of finite elements. Such a choice of the polynomials allows us to construct a piecewise polynomial basis continuous across the boundaries of elements together with the derivatives up to a given order, which is used to solve elliptic boundary value problems using the high-accuracy finite element method.

Using the Computer Algebra System SymPy to Implement the Method of Stochastization of One-Step Processes

A.V. Demidova, M.N. Gevorkyan, A.V. Korolkova, D.S. Kulyabov, L.A. Sevastianov, T.R. Velieva (Peoples' Friendship University of Russia)

When modeling phenomena such as population dynamics, the study of controlled flows, etc. there is a problem of adapting existing models to the phenomenon under investigation. To this end, we propose to obtain new models from the first principles on the basis of the method of stochastization of one-step processes. Our study has the form of an iterative process, which consists in obtaining a model and subsequently adjusting it. The number of such iterations can be extremely large. The aim of this work is to develop a software implementation of the method of stochastization of one-step processes by means of computer algebra. In this paper, we propose to use the computer algebra system *SymPy* as the basis for software implementation. Based on the developed algorithm, we obtain stochastic differential equations. The results of the program are demonstrated on the Verhulst model.

Applying Factor Analysis in Solution of Improper Problems of Linear Programming

V.V. Dikusar, N.N. Olenev (Dorodnicyn Computing Center, Federal Research Center "Computer Science and Control" of RAS), *A. Yatsko* (Technical University of Koszalin, Poland)

The improper problems of linear algebra and linear programming arise in the numerical solution of optimal control problems with mixed constraints. The regularization method of Tikhonov A.N. reduces ill-posed problems of linear algebra to systems having a unique solution due to the introduction of additional parameters. The main difficulties the choice of the regularization parameters are associated with obtaining inverse matrices. The use of factor analysis allows us to reduce these problems to the search for eigenvalues and eigenvectors of specified matrices. It is shown that the estimation of the solution of the linear programming problem due to the continuation with respect to the parameter reduces to solving an ill-posed linear system. The problem of finding eigenvalues and eigenvectors is considered as an optimization problem in the parameter space. The spherical norm of the observation matrix acts as a functional, and the eigenvalues are parameters. This problem is solved using a parallel version of the Sobol–Statnik method (search over a uniform grid). Here, the tri-diagonal Hermitian matrix serves as the first approximation, and the parameters are given on the deviation interval near the first approximation. To accelerate the convergence of the iterative process, the method of continuation of the solutions with respect to the parameter is applied. To minimize the error, the shift of the eigenvalue spectrum of the matrix is applied. Parallel calculations on the entered parameter allow to accelerate the execution of the numerical Jacobi method when continuing with respect to the parameter.

Strongly Consistent Finite Difference Approximations to Systems of PDEs

V.P. Gerdt (Joint Institute for Nuclear Research, Dubna)

In the talk we present some of our recent results obtained in collaboration with Yu.Blinkov, D.Robertz, P.Amodio and R. La Scala on computer algebra based application to study the consistency of finite difference approximations to systems of partial differential equations of the form

$$f_1 = \cdots = f_p = 0\,,$$

where $F := \{f_1, \ldots, f_p\} \subset \mathcal{R}\{u^{(1)}, \ldots, u^{(m)}\}$ is a set of partial differential polynomials with differential indeterminates (dependent variables) $u^{(1)}, \ldots, u^{(m)}$ and mutually commuting derivations $\delta_1, \ldots, \delta_n$. $\mathcal{R}$ is assumed to be the differential field of rational functions in the independent variables $x_1, \ldots, x_n$ with rational coefficients and may also include finitely many constants (parameters).

Application of Computer Algebra Methods for Investigation of the Dynamics of Axisymmetric Satellites

S.A. Gutnik (Moscow State Institute of International Relations), *V.A. Sarychev* (Keldysh Institute of Applied Mathematics of RAS)

With the help of computer algebra methods the properties of a non-linear algebraic system that determines the stationary motions of a axisymmetric satellite moving along a circular orbit subject to gravitational and active damping moments were investigated. The main attention is paid to the study of the conditions for the existence of stationary satellite motions. The computer algebra method based on the algorithm for the construction of a Gröbner basis applied to reduce the satellite stationary motion system of six algebraic equations with six variables to a single algebraic equation with one variable that determines all

stationary motions of the satellite. A classification of domains with an equal number of stationary solutions is carried out using algebraic methods for constructing discriminant hypersurfaces. The effectiveness of various algorithms for constructing Gröbner bases for the solution of the problem under consideration was compared.

Homogeneous Almost Primitive Elements of Free Algebras of Schreier Varieties

A.V. Klimakov, A.A. Mikhalev (Moscow State University)

A variety of algebras is said to be Schreier if any subalgebra of a free algebra of this variety is free. A subset of nonzero elements of a free algebra is said to be primitive if there is a set of free generators of algebra that contains it. A nonzero element of a free algebra is said to be almost primitive if it is not a primitive element of the algebra, but is a primitive element of any proper subalgebra that contains it. In this paper we consider almost primitive elements of free nonassociative algebra, free (anti)commutative algebra and free Lie algebra.

Splitting Permutation Representations of Finite Groups by Means of Gröbner Bases

V.V. Kornyak (Joint Institute for Nuclear Research, Dubna)

The approach to constructive formulation of quantum formalism proposed in *Kornyak V. V.* "Classical and Quantum Discrete Dynamical Systems" (Phys. Part. Nucl. 2013, 44, No 1) is based on the replacement of a unitary group acting in a Hilbert space over the complex field $\mathbb{C}$ by a unitary representation of a finite group in a Hilbert space over a cyclotomic field. Since any representation of a finite group can be obtained as a subrepresentation of some permutational representation, it is useful to be able to decompose an arbitrary permutation representation into irreducible components. We describe here an algorithm for splitting a permutation representation into irreducible components based on an easily computed basis of invariant bilinear forms, and using the fact that forms in invariant spaces, if properly normalized, are operators of projection into these subspaces. An important element of the algorithm is the calculation of the Gröbner basis of a zero-dimensional ideal. The algorithm is illustrated by a concrete example.

Necessary Conditions for the Existence of Algebraic Integrals of ODEs

Ya.Yu. Kuziv, M.D. Malykh, L.A. Sevastianov (Peoples' Friendship University of Russia)

Investigations of M. N. Lagutinski on the theory of integration of the differential equations were interrupted his tragic death in 1915. In our talk on the Moscow conference "Computer Algebra" in 2016 the ideas of Lagutinski are considered from viewpoint of modern computer algebra and the package Lagutinski for CAS Sagemath was presented. In this talk we use Lagutinski method for the searching of necessary conditions for the existence of algebraic integrals. For a ring R with differentiation D and basis $B = \{\phi_1, \phi_2, \dots\}$ the sequence of determinants is entered, they are called further as Lagutinski determinants. If one of Lagutinski determinants is equal to zero, the rational integral exists, moreover, we can always calculate this integral. The converse is proved for the polynomial rings. This fact can be used not only for finding the integrals but also for proving the conditions of the existence or the nonexistence of algebraic integrals. The differentiation D will be called as contracting differentiation if there is a basis in which

$$D\phi_i = c_i\phi_i + o(\phi_i), \quad c_i \in c(R).$$

For the differentiation it is possible to write out simple necessary criterion of existence of integrals: among indexes c_i there are equal. We use this statement for the investigation of integrability of Brio and Bouquet differential equation.

An Algorithm for Computing Residue Integrals for a Class of Systems of Algebraic Equations

A.A. Kytmanov, A.M. Kytmanov, E.K. Myshkina (Siberian Federal University, Krasnoyarsk)

We present an algorithm for computing residue integrals for a special class of systems of nonlinear algebraic equations. Under certain conditions such integrals coincide with power sums of the inverses to the roots of a system of equations. This makes it possible to develop elimination methods for this class of systems. We present an example of computing power sums for a particular system of equations.

Symbolic Calculations in Ising Models

V.V. Marshalov (Russian Presidential Academy of National Economy and Public Administration),
E.A. Teveleva, N.P. Tretyakov (Russian State Social University)

In this work some examples of computer algebra calculations of partition functions of Ising models, including in the case of nonzero magnetic field are given. Various algorithms for finding partition functions of two- and three-dimensional Ising models (with square and cubic lattices) are considered. The values obtained as a result of calculations using exact and approximate formulas are compared. The Onsager formula for the partition function of the two-dimensional model was calculated using some computer algebra systems. It is verified experimentally that the Onsager formula is divergent with exact calculations (including Kaufmann formulas) for small lattice sizes. Symbolic analytical systems have extremely limited application in the study of lattice spin models in the computational aspect, in view of the exponential growth of complexity with increasing system sizes. Monte Carlo imitation methods are effective here (especially when using video cards). However, as the examples examined in this paper have shown, analytical methods are indispensable when trying to obtain new exact solutions, when analyzing existing analytical solutions, and when searching for new effects. For example, in this paper we have discovered the existence of singularities on some graphs of the logarithms of the partition function of three-dimensional Ising models with nonzero magnetic field. Finding out whether this is a computational artifact or talking about the possibility of phase transitions requires further research.

Irreducible Differential Systems and Satellite Unknowns

A.A. Panferov (Moscow State University, Dorodnicyn Computing Center, Federal Research Center "Computer Science and Control" of RAS)

We consider satellite unknowns in irreducible differential systems with selected unknowns. We prove that in such systems any unselected unknown is always satellite for any set of selected unknowns. We also present a system factorization algorithm. This algorithm has polynomial complexity, but also it has some strict precondition, so it cannot be applied to every differential system.

Search for Hypergeometric Solutions of q-Difference Systems by Resolving Sequences

A.A. Ryabenko (Dorodnicyn Computing Center, Federal Research Center "Computer Science and Control" of RAS)

An algorithm to search for hypergeometric solutions of systems of linear homogeneous q-difference equations with polynomial coefficients is presented. The algorithm is based on use of the resolving sequences.

Intelligent methods for grammatically correct text synthesis in Russian

T.M. Sadykov, T.A. Zhukov (Plekhanov Russian University of Economics)

The problem of synthesis of a grammatically correct text in a natural language is known to be highly complex. The purpose of work is to develop and implement web-service that provides full algorithmic coverage of Russian language and has ability to generate grammatically correct text from any exact data for any topic. As a result, this work presents the main algorithms and features of their implementation used in the website passare.ru, which is aimed at automatic processing of linguistic data and text synthesis.

Looking for Points on a Smooth Cubic Hypersurface

A.V. Seliverstov (Institute for Information Transmission Problems of RAS (Kharkevich Institute))

It is well known that a smooth projective cubic hypersurface of dimension two or higher with a marked point over a field of characteristic zero is unirational over the field. Consequently, the set of points of the hypersurface over the field is dense in the Zariski topology. There is considered the computational complexity of the search for such points. It is shown that a dominant rational map from a projective space to the hypersurface can be calculated by a probabilistic algorithm that works without errors and completes the work with a high probability, making a polynomially bounded number of arithmetic operations over the field. In the general case, the image of the rational map does not contain all points over the field, but only a large set of such points. In particular, the calculation of such points over a finite extension of the field of rational numbers allows us to abandon the approximation of real or complex numbers, but use more possibilities of symbolic computations. The problem is closely related to the proof of the smoothness of the hypersurface and can be used to solve some combinatorial problems. Earlier, I.V. Latkin and the author have shown the set partition problem can be reduced to the problem of finding singular points of a cubic hypersurface. Rational parametrization of surfaces is used in computer graphics.

References

[1] *S. Abramov, L. Sevastyanov.* The Conference “Computer Algebra” in Moscow. ACM Communications in Computer Algebra, Vol. 50, No. 2, Issue 196, June 2016, p.61-68.