The Sixth Conference "Computer Algebra" in Moscow

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Foreword

The International Conference "Computer Algebra" was held in Moscow, Russia on June 23–25, 2025 and the conference web-site is

http://www.ccas.ru/ca/conference .

Co-organized by the Federal Research Center "Computer Science and Control" of Russian Academy of Sciences, Peoples' Friendship University of Russia and Plekhanov Russian University of Economics, the conference was devoted to inspiring discussions on computer algebra and related topics. Researchers from different countries presented talks on their latest research work.

This is the sixth edition of the conference, and the previous five were in 2016, 2017, 2019, 2021 and 2013, respectively.

1 Invited Talk Abstracts

Deciding summability via residues in theory and in practice

C.E. Arreche (The University of Texas at Dallas, Department of Mathematical Sciences, 800 W Campbell Road, Richardson, TX 75080, USA)

In difference algebra, summability arises as a basic problem upon which rests the effective solution of other more elaborate problems, such as creative telescoping problems and the computation of Galois groups of difference equations. In 2012 Chen and Singer introduced discrete residues as a theoretical obstruction to summability for rational functions with respect to the shift and q-dilation difference operators. Since then analogous notions of discrete residues have been defined in other difference settings relevant for applications, such as for Mahler and elliptic shift difference operators. Very recently there have been some advances in making these theoretical obstructions computable in practice.

Gröbner bases for investigating and solving polynomial and differential systems

Yu.A. Blinkov (Saratov State University, 83 Astrakhanskaya St, Saratov, 410012, Russian Federation)

This paper explores the theory and applications of Gröbner bases in polynomial and differential systems, emphasizing their role in solving ideal membership problems, analyzing differential equations, and constructing numerical schemes. The computational framework leverages the Python library PyGInv (a lightweight variant of GInv), integrated with SymPy, to demonstrate efficiency in symbolic computations.

Integrable cases of the Euler–Poisson equations

A.D. Bruno (Keldysh Institute of Applied Mathematics, Miusskaya Sq. 4, Moscow, 125047, Russian Federation), A.B. Batkhin (1. Keldysh Institute of Applied Mathematics, Miusskaya Sq. 4, Moscow, 125047, Russian Federation; 2. Technion – Israel Institute of Technology, Technion City, 3200003 Haifa, Israel)

In the classical problem of the motion of a rigid body around a fixed point, described by the Euler-Poisson equations, we make an attempt to connect integrability of the system with its properties near its stationary points. The method gives all known cases of integrability and several new.

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2 Contributed Talk Abstracts

Matrices of infinite series: checking non-singularity based on truncations

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Square matrices over a ring of formal power series are considered. The matrix entries (series) are given "approximately": for each of them, only a finite number of initial terms are known, and these numbers may not coincide for different matrix entries. Issues related to the possibility of guaranteeing the non-singularity of a matrix for which only this kind of "approximate" representation is known are discussed.

Formal stability investigation in Hamiltonian systems with resonances

A.B. Batkhin (1. Keldysh Institute of Applied Mathematics, Miusskaya Sq. 4, Moscow, 125047, Russian Federation; 2. Technion – Israel Institute of Technology, Technion City, 3200003 Haifa, Israel), Z.Kh. Khaydarov (Samarkand State University named after Sh.Rashidov, 140103 Samarkand, Uzbekistan)

A method for studying formal stability of equilibrium points in multiparameter Hamiltonian systems with three degrees of freedom is proposed in the case of resonances present. We detail a technique for the symbolic computation of resonance conditions, employing computer algebra and power geometry to represent the resonant variety as a rational algebraic curve and provide its polynomial parametrization.

Parametric expansions of an algebraic variety near its singularities

A.D. Bruno (Keldysh Institute of Applied Mathematics, Miusskaya Sq. 4, Moscow, 125047, Russian Federation), A.A. Azimov (Department of Algebra and Geometry, Samarkand State University named after Sh. Rashidov, Samarkand 140104, Uzbekistan)

In works [A.D. Bruno, A.A. Azimov, Parametric expansions of an algebraic variety near its singularities, Axioms 12 (2023), 469; A.D. Bruno, A.A. Azimov, Parametric expansions of an algebraic variety near its singularities II, Axioms 13 (2023), 106] parametric expansions near 5 singular points and 3 curves consisting of singular points were computed for a two-dimensional algebraic variety Ω. This paper presents general methods for computing expansions of the variety near its singularities.

Block Fermat numbers in modular arithmetic

B. Chen (University of Waterloo, Waterloo, Canada), E. Zima (University of Waterloo, Waterloo, Canada)

We show how to select balanced in size moduli of the form $2^n + 1$ to achieve accelerated modular reduction and reconstruction. Experimental implementation demonstrates that this choice of moduli provides better performance of modular arithmetic compared to the popular moduli of the form $2^n - 1$.

Non-minimality of minimal telescopers explained by residues

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Elaborating on an approach recently proposed by Mark van Hoeij, we continue to investigate why creative telescoping occasionally fails to find the minimal-order annihilating operator of a given definite sum or integral. We offer an explanation based on the consideration of residues.

On relative primality and other properties of trinomials

R. Dougherty-Bliss (Dartmouth College, Hanover, NH, USA), M. Kobayashi (Dartmouth College, Hanover, NH, USA), N. Ter-Saakov (Rutgers University, New Brunswick, NJ, USA), E. Zima (University of Waterloo, Waterloo, Canada)

We discuss some properties of trinomial polynomials. These can be used to generate new balanced-size sets of moduli for accelerated modular arithmetic.

On the integrability of an ODE system with an inhomogeneous right-hand side

V.F. Edneral (Skobeltsyn Institute of Nuclear Physics of Lomonosov Moscow State University, SINP MSU, 1(2), Leninskie gory, GSP-1, Moscow 119991, Russian Federation)

The report describes the application of the normal form method to the study of a polynomial modification of the Liénard equation with a 4th-order nonlinearity. A two-dimensional set of parameter values is found for which the corresponding system has a first integral.

The dual quaternion algebra and its implementation in Asymptote language

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The algebras of dual quaternions and screws are often opposed to geometric algebra. The purpose of this paper is to describe the algebra of dual quaternions and the algebra of screws, to give a number of examples of the use of dual quaternions to describe the screw motion of points, lines and planes in three-dimensional space. This algebra is very poorly covered in the literature, and the actively used principle of Kotelnikov-Study transfer is apparently forgotten. All calculations were performed using the Asymptote language. Structures were created that implement dual numbers, quaternions, and dual dual quaternions, as well as a set of computational tests to verify these structures.

Investigation of the periodic planar oscillations of a two-body system in an elliptic orbit using the computer algebra methods

S.A. Gutnik (1. MGIMO University, 76, Prospekt Vernadskogo, 119454, Moscow, Russian Federation; 2. Moscow Institute of Physics and Technology, Institutskii per. 9, Dolgoprudny, 141701 Russian Federation)

Computer algebra methods are used to investigate the planar oscillations of a system of two bodies connected by a spherical joint that moves along an elliptic orbit under the action of gravitational torque in the plane of the orbit. It is shown that the motion of the two-body system is described by periodic planar oscillations. All the relevant symbolic computations are performed with the help of computer algebra systems.

Searching for an imperfect palindrome

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We consider algorithm for optimizing an imperfect palindrome in the input sequence. Our algorithm runs in quadratic time, i. e., it is faster than exhaustive search of admissible palindromes. Also we propose new approach called "trimming" to find long substrings of a sequence, that are closer to palindrome, than sequence itself.

Quantum mechanics through the lens of finite groups: computer algebra insights

V.V. Kornyak (Joint Institute for Nuclear Research, 6 Joliot-Curie St, Dubna, 141980, Russian Federation)

The use of non-constructive infinities in physical theories can lead to contradictions and non-physical artifacts. Quantum behavior can be fully described using only finite subgroups of the general unitary group — specifically, the Weyl–Heisenberg group and its extension, the Clifford group. By formulating quantum theory in terms of these groups, we completely eliminate the need for the continuous unitary group, which leads to important empirical consequences. Crucially, this approach provides a natural explanation for the observed absence of quantum entanglement and interference between distinct types of elementary particles. Moving away from continuum-based mathematics also requires redefining the concept of quantum states: the continuous projective Hilbert space should be replaced by some combinatorial structure. Using computer algebra calculations, we study a potential framework for building constructive quantum states, governed by a fixed set of physically motivated criteria.

Moshinsky atom as a test for FEM on hypercubes

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Third- and fourth-order FEM schemes with multivariate Hermite interpolation polynomials of a d-dimensional hypercube for solving boundary value problems (BVPs) on hyperparallelepipedal meshes are elaborated. An exactly solvable model of a system of several identical particles with pair oscillator interaction known as the Moshinsky atom is used as a test example. To describe the energy spectra of symmetric and antisymmetric bound states, the 2-, 3-, 4-, and 5- dimensional BVPs with Dirichlet and Neumann boundary conditions on a nonrectangular domain are formulated. To generate new FEM schemes with mixed partial derivatives, additional affine coordinate transformations are applied. Benchmark calculations of the BVPs confirm the order of declared FEM schemes.

Existence of Liouvillian solutions in the problem of motion of a heavy gyrostat under the action of gyroscopic forces

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We study the problem of motion of a gyrostat about a fixed point under the action of gravity and gyroscopic forces in the Hess integrability case. It is shown, that the solution of the problem is reduced to the integration of the second order linear differential equation with rational coefficients. Using the Kovacic algorithm, we obtain the conditions on the parameters of the problem under which we can find the general solution of the corresponding second order linear differential equation in explicit form.

Symbolic investigation of a difference scheme for wave processes in coaxial elastic cylindrical shells filled with an incompressible viscous fluid

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This study investigates difference schemes for modelling wave processes in coaxial elastic cylindrical shells filled with an incompressible viscous fluid. Using symbolic computation methods (Gröbner bases) and numerical methods, the authors derive a Crank–Nicolson-type scheme for a coupled nonlinear system of equations analogous to the Korteweg–de Vries (KdV) equation. The first differential approximation method is employed to analyze the scheme's accuracy, supported by exact soliton solutions and numerical experiments.

Combinatorics of Lyndon-Shirshov words and algorithms of symbolic computation in Lie superalgebras

A.A. Mikhalev (Faculty of Mechanics and Mathematics, M.V. Lomonosov Moscow State University, Leninskie Gory, Moscow, 119234, Russian Federation)

We consider main combinatorial properties of Lyndon-Shirshov words and their applications in algorithms of symbolic computation in free Lie (p-)superalgebras and in Lie (p-)superalgebras given by generators and defining relations.

FEM calculations of Coulomb two center problem

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An algorithm for solving of the boundary value problem for the discrete spectrum of a two-center Coulomb system is presented. The energy and separation constant eigenvalues and the corresponding eigenfunctions are calculated by the secant method on a suitable grid of the parameter, the distance between two Coulomb charges. The eigensolutions at each step of the secant method are calculated using KANTBP 5M program, which implements the finite element method in Maple.

Symbolic computing in the constructing problem of controlled compartmental model

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The results of developing a toolkit that enables symbolic computations in the process of solving problems of constructing controlled compartmental models of dynamic systems are presented. A controlled compartmental of epidemiology model SIRHU which is a generalization of the SIRH model to a controlled case is constructed and studied. Control actions are specified in the form of rules, the transition intensity in which is a changing parameter. When constructing this controlled model the implementation of a domain-specific language is used to construct models based on the interaction scheme between compartments and on the developed algorithm for the simulation implementation of models taking into account control. Computational experiments are conducted to simulate a controlled system of epidemic spread related to the SIRHU type. The analysis of the trajectory dynamics of the simulation model and the corresponding differential model is performed. The obtained results can be used to solve problems of modelling dynamic processes based on one-step interactions.

Symbolic computation in studying the dynamics of two-body system with variable masses

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We discuss symbolic calculations in studying the classical two-body problem with variable masses. The Gyldén equation describing relative motion of the bodies is not integrable, in general, and so the problem is solved in the framework of the perturbation theory. Two approaches to reduce the Gyldén equation to the form appropriate for application of the perturbation methods are discussed. It is shown that although differential equations of the perturbed motion are different the both models demonstrate similar behavior of the secular perturbations of the orbital elements. All the relevant symbolic calculations are performed with the computer algebra system Wolfram Mathematica.

The generic-case complexity of finding a binary solution to a system of linear equations

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Using the Schwartz–Zippel lemma, we estimate the generic-case complexity of finding a binary solution to a system of linear equations over an infinite field. Compared to previous works, the new algorithm

is applicable to systems with a smaller number of linear equations, but it requires more running time. The algorithm works over a field of any characteristic. If the system has many binary solutions, then our algorithm is not applicable. So, in the worst case, there is no polynomial upper bound on the computational complexity.

On the summation of Fourier series over the roots of transcendental equations using annihilation

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Trigonometric series in the roots of transcendental equations are considered. Similar series arise in problems of mathematical physics in the case of boundary conditions of the third kind. For this class of series, sufficient conditions for their representability in the finite form of elementary functions are indicated. We generalize to this case our previous works based on the use of finite expressions for divergent Fourier series. In this case, we obtain a simple method for determining the elementarity of the ζ-function of N.S. Koshlyakov and its generalizations. These considerations are used to create new functions of symbolic summation in computer algebra systems.

On the determination of the volumetric heat capacity and the thermal conductivity of a substance in the three-dimensional case

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The problem of simultaneous identification the temperature-dependent thermal conductivity and volumetric heat capacity of the investigated substance in the three-dimensional case is considered. The consideration is based on the first boundary value problem for a three-dimensional unsteady heat equation. The inverse coefficients problem was reduced to a variational problem. The optimization problem was solved using gradient methods of functional minimization. Based on the fast automatic differentiation technique exact values of the cost functional gradient components were obtained.
