On Solutions of Linear Functional Systems (Errata)

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The bounds given in the paper are valid when the system of recurrences (5) and the equations (6) are valid for all $n \in \mathbb{Z}$, which is the case when using the power basis $\mathcal{P} = \langle x^n \rangle_{n \geq 0}$, because it can be extended to negative values of n. Therefore the bounds given in the paper are valid for differential and q-difference equations provided that the basis $\mathcal{P}$ is used to produce the recurrence.

In the case when the basis used is valid only for $n \geq \mu$ for some $\mu \in \mathbb{Z}$ (for example we can have $\mu = 0$ for difference equations), then the system of recurrences (5) and the equations (6) are valid only for $n \geq \mu$. Since we apply (6) to $n = N - s$ in the proof of Theorem 4, that proof is valid only when $N - s \geq \mu$, i.e. $N \geq s + \mu$. Therefore, the correct version of Theorem 4 is the following, where $\deg(0) = -\infty$ by convention:

Theorem 4 *Let L be an $r \times m$ matrix with entries in $End_{\mathcal{B}}(K[x])$, $F \in K[x]^r$, $Y \in K[x]^m$ be nonzero and $N = \max_i\{\deg Y_i\}$. If $LY = F$ then either $N \leq s + \max\{\mu - 1, \max_i\{\deg(F_i)\}\}$ or $Ker(M_s(N-s)) \neq 0$, where M_s is as in (6) and μ is either $-\infty$ or an integer such that the equations (6) are valid only for $n \geq \mu$.*

When the basis $\mathcal{P}$ is used, then the transformed recurrences remain valid for all $n \in \mathbb{Z}$ and the bounds in the paper are valid. Otherwise, for example when computing recurrences from difference equations, the value of μ in the above theorem can change when transforming the recurrence as described in Section 4: initially $\mu = 0$ and the lower bounds for each row of (5) are $n_1 = \ldots = n_r = 0$. When the algorithm replaces row i_0 by $(\phi^{-1}w_1, \ldots, \phi^{-1}w_m)$ with $w = v^T R$, then n_{i0} must be replaced by $1 + \max_{i|v_i \neq 0}\{n_i\}$. Throughout the algorithm, row i of (5) is valid for $n \geq n_i$, so when we produce a nonsingular trailing matrix, we have $\mu = \max_i\{n_i\}$. By the above theorem, the correct bound on the degree of the polynomial solutions at the end of the process is

$$N \leq s + \max\{\max_i\{n_i\} - 1, \max_i\{\deg(F_i)\}\} \quad \text{or} \quad \mathrm{Ker}(M_s(N-s)) \neq 0\,.$$

where M_s is the nonsingular trailing matrix at the end of transformation.