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The Research Seminar on Computer Algebra in 2008–2009

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Abstract—An annual report on meetings of the scientific research seminar on computer algebra is presented.

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1. ABOUT THE SEMINAR

The subjects of the seminar are new results in computer algebra; more precisely, these are symbolic algorithms, their implementation, and related topics in system programming.

In the academic year of 2008–2009, the meetings were usually held monthly (every third Wednesday) at the Department of Computational Mathematics and Cybernetics of Moscow State University and at the Skobeltsyn Institute of Nuclear Physics. The Web site of the seminar at the address http://theory.sinp.msu.ru/dokuwiki/docu.php/calg.main contains information about the scheduled and already presented papers.

In the previous years, meetings in May were traditionally organized in cooperation with the Laboratory of Information Technologies of the Joint Institute for Nuclear Research (JINR) and were held in Dubna at JINR. In 2009, the organizing committee of the international conference on mathematical modeling and computational physics (MMCP'2009), which was held in July at JINR, included a computer algebra section into the conference program. Therefore, the traditional joint meeting of the seminar in Dubna was not held separately but constituted a part of the conference section (see Section 3).

2. REGULAR MEETINGS

Below is the list of presentations from September through April.[1]

S.A. Abramov (Computing Center, Russian Academy of Sciences; sergeyabramov@mail.ru). *On Entire Solutions of Linear Difference Equations with Polynomial Coefficients.*

It is well known that any linear difference equation of order d with polynomial (over the field of complex numbers $\mathbb{C}$) coefficients has a fundamental system of entire solutions (C. Praagman, 1986). It is shown in the paper that this result can be strengthened. Namely, the $\mathbb{C}$-linear space of sequences that are restrictions of the entire solutions of an equation of order d to $\mathbb{Z}$ has dimension d. It is also shown that a basis for this space of sequences can be found algorithmically.

The results were obtained jointly with M. Barkatou, M. Petkovsek, and M. van Hoeij.

V.V. Kornyak (JINR, Dubna; kornyak@jinr.ru). *Extended Wreath Products.*

Let G and Γ be groups acting on sets X and Ψ, respectively. Explicit formulas for a sequence of multiplications on the Cartesian product $\Gamma^X \times G$, together with actions on the set of functions Ψ^X, are presented. These structures form an infinite family of semidirect products of Γ^X and G parameterized by two integers $m \in \{-1, 0\}$ and $k \in \mathbb{Z}$. In this family, the standard direct and wreath products correspond to the pairs $(m, k) = (0, 0)$ and $(m, k) = (-1, -1)$, respectively. These constructs are used in studies of discrete dynamic systems with symmetries by methods of computer algebra and computational group theory.

A.A. Gusev and O. Chuluunbaatar (JINR, Dubna; gooseff@jinr.ru, chuka@jinr.ru). *Asymptotic Expansions of Solutions to Two-Dimensional Elliptic Boundary Value Problem.*

A three-dimensional boundary value problem for an elliptic partial differential equation with an axial symmetry (the Schrödinger equation with Coulomb and transverse oscillator potentials) is reduced to a

[1] The list of presentations in the years of 1995–2008 can be found in [1–13].

two-dimensional one. By using a basis of angular functions, the two-dimensional boundary value problem is reduced to a system of second-order ordinary differential equations with homogeneous boundary conditions of the third type at the boundary points of a finite interval of the radial variable. Recurrence relations for calculating asymptotic expansions of the desired solution to the boundary value problem for small and large values of the radial variable and for setting boundary conditions are derived with the help of the Maple system.

N.K. Zaytsev (Gubkin State University of Natural Oil and Gas; ionomer@ionomer.ru). *On Some Problems of Applied Chemistry.*

A survey of chemical kinetics problems that could be solved by computer algebra methods is given.

S.Yu. Vernov (Skobeltsyn Institute of Nuclear Physics, Moscow State University; syvernov@mail.ru). *Construction of Exact Solutions for Nonlocal Cosmological Models.*

Cosmological models driven by a nonlocal scalar field, which were inspired by the string field theory, are studied. A distinctive feature of these models is that the equations of motion are linear in the Minkowski space-time. In a non-flat space-time, the Einstein equations are nonlinear; however, the equations of motion are linear in the scalar field. This property makes it possible to localize the model for searching special solutions. The considered linear nonlocal model is equivalent to an infinite number of local models. Exact special solutions of the nonlocal Einstein equations in the Friedmann metrics are found with the help of the Maple system. One of these solutions describes a monotonically increasing Universe with the phantom dark energy.

S.D. Makhortov (Voronezh State University; sd@expert.vn.ru). *Multilevel Minimization of Conditional Rewrite Systems.*

Algebraic lattice-based models of conditional equational theories of term rewrite systems are presented.

S.A. Abramov (Computing Center, Russian Academy of Sciences; sergeyabramov@mail.ru). *On an Undecidable Problem Related to Difference Equations.*

The paper is published in this issue of the journal.

V.F. Edneral (Skobeltsyn Institute of Nuclear Physics, Moscow State University; edneral@theory.sinp.msu.ru). *On the Integrability of a Plane ODE System near a Degenerate Stationary Point.*

An autonomous system of ordinary differential equations resolved with respect to derivatives is considered. To study local integrability of the system, an approach based on the power geometry method and on the computation of the resonant normal form is used. For a 5-parametric system of this kind, necessary and sufficient conditions on the parameters for which the system is locally integrable near a degenerate stationary point are found.

The results were obtained jointly with A.D. Bruno (Keldysh Institute for Applied Mathematics, Russian Academy of Sciences).

A. Aranson (Scientific–Research Institute of Long-Range Radio Communication, Moscow; aranson@cbgrad.ru). *Analysis of ODE Systems by Algorithms of Power Geometry in CAS Maxima.*

Power geometry algorithms for calculating asymptotic expansions of solutions of ODE systems, as well as a collection of functions developed by the author that implement these algorithms in the computer algebra system Maxima, are considered. These functions were tested in the computation of known asymptotic expansions of solutions of the autonomous Henon–Heiles ODE system. Results of application of these functions to the calculation of asymptotic expansions of solutions of the non-autonomous Kowalewski ODE system are discussed.

3. COMPUTER ALGEBRA SECTION OF THE INTERNATIONAL CONFERENCE MMCP'2009

Below is the list of papers presented on the computer algebra section of the conference:

S. Fritzsche (Institute of Physics, Heidelberg University; s.fritzsche@gsi.de). *A Computer-Algebraic Approach to the Simulation of Multi-Qubit Systems.*

V.P. Gerdt (JINR, Dubna; gerdt@jinr.ru), R. Kragler (Weingarten University of Applied Sciences; kragler@hs-weingarten.de), and A.N. Prokopenya (Brest State University; prokopenya@brest.by). *Implementation of Some Algorithms for Quantum Computation with Mathematica.*

V.G. Romanovski (Maribor University, Slovenia; valery.romanovski@uni-mb.si). *Applications of Computational Algebra Algorithms to Qualitative Theory of Differential Equations.*

O.V. Tarasov (JINR, Dubna; otarasov@jinr.ru). *Functional Equations for Feynman Diagrams.*

V.P. Gerdt (JINR, Dubna; gerdt@jinr.ru). *On Decomposition of Nonlinear Differential Systems into Involutive Subsystems.*

D. Stefanescu (Bucharest University; stef@rms.unibuc.ro) and S.A. Evlakhof (JINR, Dubna; evlakhov@jinr.ru). *Complexity of Bounding Polynomial Roots.*

V.V. Kornyak (JINR, Dubna; kornyak@jinr.ru). *Symmetries, Gauge Invariance, and Quantization in Discrete Models.*

S.A. Abramov, D.E. Khmelnov (Computing Center, Russian Academy of Sciences; sergeyabramov@mail.ru, dennis_khmelnov@mail.ru), and M.A. Barkatou (Limoges University, France; moulay.barkatou@unilim.fr). *Interlacing Solutions of Linear Difference Equations.*

A.S. Surzhykov and S. Fritzsche (Institute of Physics, Heidelberg University; surz@physi.uni-heidelberg.de, s.fritzsche@gsi.de). *DIRAC: Computer-Alge-*

bra Tool for Studying the Structure and Dynamics of High-Charge Heavy Ions.

A.M. Raportirenko (JINR, Dubna; ram@sunct1.jinr.ru). *Common LISP-based REDUCE.*

N.N. Vasiliev (St. Petersburg Division of the Steklov Institute of Mathematics, Russian Academy of Sciences; vasiliev@pdmi.ras.ru) and V.O. Nikolaenko (St. Petersburg State University; itsmyway.leranik@gmail.com). *Enumeration of Permutation Binomials over Finite Fields.*

I. Dimovski and M. Spiridonova (IMI, Bulgarian Academy of Sciences; dimovski@math.bas.bg, mspirid@math.bas.bg). *An Implementation of the Heaviside Algorithm.*

A.D. Bruno (Keldysh Institute for Applied Mathematics, Russian Academy of Sciences; bruno@keldysh.ru) and V.F. Edneral (Skobeltsyn Institute of Nuclear Physics, Moscow State University; edneral@theory.sinp.msu.ru). *On Integrability of a Planar ODE System near a Degenerate Stationary Point.*

I.V. Puzynin, T.P. Puzynina, and V.T. Thach (JINR, Dubna; ipuzynin@jinr.ru, puzynina@jinr.ru, votrongthach@jinr.ru). *SLIPM—a MAPLE Code for the Numerical Solving of Sturm–Liouville Partial Problem Based on the Continuous Analog of Newton's Method.*

O.N. Pereslavtseva (Tambov State University; Pereslavtseva@rambler.ru). *Calculation of Characteristic Polynomials for Matrices: Serial and Parallel Algorithms.*

N.A. Malaschonok (Tambov State University; nmalaschonok@narod.ru). *An Algorithm for Symbolic Solving Systems of Partial Differential Equations.*

M.G. Kokotchikova, D.S. Kulyabov, and L.A. Sevastianov (People's Friendship University, Moscow; mgkokotchikova@gmail.com, yamadharma@gmail.com, leonid.sevast@gmail.com). *Computer Implementation of a Regularized Method for Wavefront Set Recovering by Results of the Hartmann Test with the Help of Analytical Calculations in Axiom CAS.*

S.D. Makhortov (Voronezh State University; sd@expert.vn.ru). *Multilevel LP-Structures in Rewriting Systems.*

V.V. Malanin and I.E. Poloskov (Perm' State University; rector@psu.ru, Igor.Poloskov@gmail.com). *Symbolic–Numeric Schemes for Analysis of Deterministic and Stochastic Systems with Aftereffect.*

D.A. Yanovich (JINR, Dubna; yan@jinr.ru). *Reduction-Level Parallel Computations of Gröbner and Janet Bases.*

Yu.A. Blinkov (Saratov State University, BlinkovUA@info.sgu.ru). *On Computation of the Bifurcation Points for the Logistic Map.*

V.P. Gerdt, M.V. Zinin (JINR, Dubna; gerdt@jinr.ru, mzinin@gmail.com), and Yu.A. Blinkov (Saratov State University, BlinkovUA@info.sgu.ru). *Distributed and Recursive Representations in Computing Boolean Gröbner Bases.*

A.B. Aranson (Scientific–Research Institute of Long-Range Radio Communication, Moscow; aranson@cbgrad.ru). *Analysis of ODE Systems by Algorithms of Power Geometry in CAS Maxima.*

N.N Vasiliev (St. Petersburg Division of the Steklov Institute of Mathematics, Russian Academy of Sciences; vasiliev@pdmi.ras.ru) and D.A. Pavlov (St. Petersburg State University; dmirty.pavlov@gmail.com). *Combinatorics of Universal Gröbner Bases and Multidimensional Young diagrams.*

P.G. Sutyrin (Moscow State University; psutyrin@cs.msu.su). *The Sage Free Open-Source Mathematical Software in the Theory of Formal Languages.*

D.E. Khmelnov and A.A. Ryabenko (Computing Center, Russian Academy of Sciences; dennis_khmelnov@mail.ru, ryabenko@cs.msu.su). *Pattern Matching Summation in MAPLE.*

E.A. Grebenikov and N.I. Zemtsova (Computing Center, Russian Academy of Sciences; e-greben@yandex.ru, zemni@yandex.su). *New Homographic Solutions in Newton's Many-Body Problem.*

S.N. Perepechko and A.N. Voropaev (Petrozavodsk State University; persn@aport.ru, voropaev@psu.karelia.ru). *The Number of Fixed-Length Cycles in an Undirected Graph: Explicit Formulas in the Case of Small Lengths.*

V.S. Rikhvitskii (JINR, Dubna; rqvtsk@jinr.ru). *Automation of Qualitative Analysis of a Given Parameterized System of Algebraic Differential Equations with the Help of Computer Algebra Tools.*

R.M. Yamaleev (JINR, Dubna; iamaleev@servidor.unam.mx). *Evolution Equations for Roots and Coefficients of Polynomials and Related Generalized Dynamics.*

Abstracts of the papers are published in book [14].

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