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In Memory of Manuel Bronstein

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On June 6, 2005, Manuel Bronstein—a prominent scientist whose contribution to computer algebra and many other areas of mathematics and computer science can hardly be overestimated—died of a heart attack. He was only forty-one.

A truly talented man, who was endlessly devoted to science, has passed away. Manuel worked with all his strength, enthusiastically, and was always researching several difficult problems simultaneously. Everyone who knew him remembers that he was witty, extremely keen in intellect, and cheerful. When not working, he could take part in discussions on diverse topics, and his partners admired him for his sudden impromptus, jokes, felicitous remarks, and unexpected viewpoints on the many little nothings of life.

Manuel was born on August 28, 1963, near Paris. His father was a physician, and his mother was a sculptor. Having graduated from school in France, he entered Berkeley University (USA, California), where, in 1987, he defended his PhD thesis under the supervision of Professor M. Rosenlicht. For three years, he worked at the IBM Research Center; then, from 1990 to 1997, in the Swiss Federal Institute of Technology (ETH), and since 1997, in France, in the French National Institute for Research in Informatics and Automation (INRIA) in Sophia Antipolis.

The dissertation defended at Berkeley was devoted to a very difficult problem related to symbolic integration (or integration in finite terms). Although the theory of integration was developed by R. Risch (another Ph.D. student of Rosenlicht) who presented in 1968 an algorithm for integration of elementary functions, it turned out that this algorithm was far from effective. Manuel significantly improved it (in particular, by generalizing B. Trager's algorithm for algebraic functions to an algorithm for the mixed case of elementary functions). While working for IBM, he implemented the integration algorithm in the Axiom system. At that time, this was the most powerful program for integration of functions. Manuel presented results of his studies in a large article published in 1990 in the *Journal of Symbolic Computation.* Later, he intended to write a monograph in two volumes devoted to all aspects of symbolic integration. The first volume was written and went through two editions at the Springer publishing house in 1997 and 2004. The second volume remained uncompleted.

It was typical of Manuel to concentrate on urgent difficult problems. After the problem of integration, he studied the problem of searching for closed-form solutions to ordinary linear differential equations. In particular, in 1992, he designed a rather general algorithm for finding solutions in the field generated by the coefficients of the equation. In construction of these solutions, one usually proceeds from a tower of extensions of the basic field. However, the key point is the possibility of finding solutions in the basic field, which contains the coefficients. This demonstrates the excep-

tional value of this result by Manuel. Many problems of differential algebra have analogues in the difference case. It is also well known that, as a rule, these difference analogues are much more difficult to solve. Nevertheless, in 2000, Manuel developed an algorithm for searching for solutions in the field of coefficients for the case of difference equations. Moreover, he constructed a universal general algorithm that covers differential, difference, and q-difference equations as special cases. This universality was attained by considering the problem on the level of noncommutative Ore polynomials. At the same time, he significantly advanced in the development of the theory of unimonomial field extensions, whose foundations were laid by M. Karr in the early 1980s. These results allowed a number of well-known algorithms for searching for various solutions of linear ordinary equations with polynomial coefficients to be generalized to much more complicated situations.

As for the Ore polynomials, it should be emphasized that the very idea of using them in computer algebra was first proposed by Manuel (together with M. Petkovšek) in a paper published in *Programming and Computer Software* in 1994. This idea was important not only from the theoretical standpoint; it also demonstrated the possibility of designing universal computer programs adjustable to the differential, difference, and some other cases. This approach is widely used nowadays by developers of computer algebra algorithms and systems.

The aforementioned paper devoted to this universal approach is not the only publication by Manuel in *Programming and Computer Software*. In 1992, he published a survey of methods for solving ordinary differential equations and integration in this journal. With the help of this survey, many specialists actively working in related scientific areas managed to penetrate into this involved subject. In 1993, Manuel was a co-editor of a special issue of *Programming and Computer Software* devoted to computer algebra.

Far from intending to give here a complete survey of Manuel's results, we mention only that he obtained many profound and valuable results not only on integration and ordinary differential and difference equations, but also on special functions, partial differential equations, operator factorization, and reducibility of systems of equations to special forms. He also published nice works on linear algebra, algebraic geometry, etc.

Manuel was a brilliant programmer. He artistically implemented all his algorithms in a number of computer algebra systems. Recently, he actively worked on the *Aldor* system and wrote a family of computer-algebraic libraries for it, namely, the *libaldor* and *Algebra* libraries (which provide the user with basic data structures and their operation procedures that are necessary for applications of computer algebra) and the Sum^{it} library (which contains efficient programs implementing complex modern algorithms for transforming and solving linear ordinary differential and difference equations). For the Sum^{it} library, he also developed two interactive interfaces *bernina* and *shasta*, which made the functions of this library available from other computer algebra systems. These libraries and interactive interfaces are high-quality tools that are widely used in many research centers.

As noted earlier, since 1997, Manuel worked at INRIA. At this institute (his last place of work), he headed a research group consisting of first-rate specialists. Each of them worked on his or her particular scientific problem, and witnesses of his discussions with collaborators were amazed by a deep insight of Manuel into all these problems and by his ability to easily pass in these discussions from one problem to another. The intellectual virtuosity that he demonstrated in these discussions was magnificent.

Manuel was a member of Editorial Boards of some leading journals and scientific series, for instance, the *Journal of Symbolic Computation* and the series *Algorithms and Computation in Mathematics*. He was a member of the program and organizing committees of several respected conferences and often chaired these committees. This particularly relates to the annual international ISSAC conference. He was also a vice-president of SIGSAM, the international group on symbolic and algebraic manipulation. In this role, he proposed and realized many fruitful ideas. For instance, for the ISSAC'05 conference, which was held in July of 2005, he had prepared a CD that contained not only texts of all the talks given at the conference but also some new software and other information valuable for everyone interested in computer algebra and its applications. Unfortunately, he was not to take part in that conference. That CD was distributed to all the participants of the conference and will remind them of Manuel.

He participated fruitfully in international research projects. For instance, in the 1990s, he was one of the leaders of the European projects Cathode 1 and Cathode 2 devoted to computer-algebraic methods for solving ordinary differential equations. During the last ten years, Manuel co-headed some projects involving Russian scientists, namely, "Computer algebra and linear functional equations" (RFBR–INTAS), "Direct computer-algebraic methods for explicit solution of systems of linear functional equations" (French—Russian Lyapunov Center), and "Computer algebra and (q-) hypergeometric terms" (Eco-Net program of the French Ministry of Foreign Affairs). His last voyage abroad was to Russia on May 15–19, 2005, within the framework of the Eco-Net program.

It should be noted that Manuel was particularly interested in Russia and events there. It is appropriate to mention that his father's family had Russian roots and Manuel himself had chosen Russian as the foreign language to study at high school (he told that, on the final exam, he had to read a passage from "Second Lieutenant Kizhe" by Yu.N. Tynyanov). Later, he read scientific journals in Russian and even translated some papers.

And when he met his future wife Karola in Leipzig in 1990, the Russian language helped them to communicate, although it was not a native tongue to either.

Remembering the joint work with Manuel, we would like to mention his remarkable ability to grasp instantly mathematical ideas and the extraordinary mental agility, which followed from his acute analytical sense. If a problem that arose in a discussion at the blackboard or was proposed by somebody was of interest to Manuel, he, as a rule, immediately proposed several approaches to solving it, including quite unusual and promising ones. Having outlined these approaches, he immediately started to develop them in detail. He made some calculations on the blackboard so fast that, sometimes, it was hard to follow them. As a result of such an improvisation, either the question was completely answered or real obstacles for further investigation were found. And Manuel often performed such analyses without any intention to be a coauthor of the work. He was a benevolent man and readily gave detailed answers to questions of people whom he scarcely knew, who asked him for a consultation or advice during a break of a conference.

Of course, Manuel's scientific interests were not restricted to only difficult classical problems. Computer algebra is known to have at its disposal complete algorithms for solving a number of such problems. However, the computational complexity of these algorithms is very high, and they are hard to implement. Manuel was interested in consideration of special cases of these problems and in simplifying and refining algorithms by using heuristics and other methods. The results of his work in this area included a new version of the algorithm of parallel integration (the first versions of the algorithm of parallel integration were proposed in the late 1970s and early 1980s by A. Norman, P. Moore, and J. Davenport; here, the term "parallel" does not relate to multiprocessor execution, and Manuel suggested replacing this term with "flat integration"). In general, this algorithm is not as powerful as the complete version of the Risch—Bronstein algorithm for symbolic integration; however, it may be implemented in just a hundred lines of code. A note on the algorithm of parallel integration is published in this issue of *Programming and Computer Software.* This note is an extended abstract of Manuel's talk at the joint seminar on computer algebra of the MSU and JINR (Joint Institute for Nuclear Research) in Dubna on May 18, 2005. It was submitted for publication in *Programming and Computer Software* on June 3, three days before his death overtook him outside his hometown, in Montpellier. He went there for a few days to discuss with biologists the possibility of describing some biological models by recurrence relations of a special form. Manuel was going to try to solve these relations using an original approach he was working on in his last days. The stock of his ideas and intentions seemed to be endless…

Providing for his large family (he was the father of six children), he was always ready to support his friends, colleagues, and associates, and helped them any time when he felt that they needed his assistance or sympathy. He never stopped being friendly to people around him.

Manuel was just as benevolent and kind as he was outstandingly talented. His name and his accomplishments in computer algebra have already found their high place in sciences. His death is a grievous, irreplaceable loss for everyone who was lucky to work with him or just be acquainted with him.