Linearization of scalar ordinary differential equation

Dmitry A. Lyakhov[1], Vladimir P. Gerdt[2]

[1]Radiation Gaseous Dynamics Lab
A. V. Luikov Heat and Mass Transfer Institute, Minsk, Belarus

[2]Laboratory of Information Technologies
Joint Institute for Nuclear Research, Dubna, Russia

Seminar on Computer Algebra, November 18, 2015, Moscow, Russia

Contents

Introduction

Sophus Lie has designed explicit form of linearizability criterion for second-order ordinary differential equations. It is convenient and easy-to-use to determine whether ODE can be linearized by point transformation.

Theorem

Second-order ordinary differential equation can be linearized by point transformation if and only if it is of the form

$$y'' + F_3(x,y)(y')^3 + F_2(x,y)(y')^2 + F_1(x,y)y' + F(x,y) = 0$$

and functions satisfy

$$3(F_3)_{xx} - 2(F_2)_{xy} + (F_1)_{yy} = (3F_1F_3 - F_2^2)_x - 3(FF_3)_y - 3F_3F_y + F_2(F_1)_y,$$

$$3F_{yy} - 2(F_1)_{xy} + (F_2)_{xx} = 3(FF_3)_x + (F_1^2 - 3FF_2)_y + 3F(F_3)_x - F_1(F_2)_x.$$

Linear and linerizable equations of second order admit symmetry algebra of maximal dimension, that is equal to 8, and only them.

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Prehistory

- Sophus Lie's pioneering work on linerization
- Cartan's equivalence method
- Grebot, Ibragimov, Meleshko, Suksern obtain explicit formulas of linearizable differential equations of third and fourth order, also studied some cases of system
- Doubrov obtained explicit formulas for trivializable differential equations of arbitrary order

Question

Do we actually need an explicit expressions of linerizable equations on practice?

Answer

No, if we have an algorithm for determination linearizability and performing linearizing transformation.

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From computational point of view it is very important to reduce number of unknowns as much as possible.

Theorem

Every nonsingular linear differential equation of order $n > 2$ could be reduced to Laguerre-Forsyth canonical form

$$u^{(n)}(t) + \sum_{i=0}^{n-3} A_i(t)u^{(i)}(t) = 0,$$

by means of some point transformation.

Remark

In case $n = 2$ canonical form is

$$u''(t) = 0.$$

Thus equation is linearizable, if and only if it is trivializable.

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Objective is to apply original Lie procedure for test-linearization, but in pure algorithmic way. It means substitution of point transformation

$$u = f(x, y), t = g(x, y), J = f_x g_y - f_y g_x \neq 0$$

to canonical form of linear ordinary differential equations of n-th order, which implies

$$y^{(n)}(x) + \frac{P(y^{(n-1)}, ..., y')}{J(g_x + g_y y')^{n-2}} = 0,$$

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Algorithm

For given ODE of rational form

$$y^{(n)}(x) + \frac{M(y^{(n-1)}, \ldots, y')}{N(y^{(n-1)}, \ldots, y')} = 0,$$

the question of linearizability is equivalent to existence of functions f, g, A_i so

$$\frac{M(y^{(n-1)}, \ldots, y')}{N(y^{(n-1)}, \ldots, y')} = \frac{P(y^{(n-1)}, \ldots, y')}{J(g_x + g_y y')^{n-2}}$$

as rational functions in $(y^{(n-1)}, \ldots, y')$. It implies to big system of nonlinear partial differential equations.

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Also we must add to system differential conditions, which mean that A_i are functions only of t and inequality $J \neq 0$.

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Differential Thomas decomposition

Linearizability is equivalent to consistency of system of nonlinear partial differential equations and inequalities, and thus is rewritten in fully algebraic terms. In order to check consistency we are using differential Thomas decomposition (Gerdt'2012)

- It splits system into disjoint set of simple, square-free, involutive subsystem
- If it is inconsistent, then decomposition is empty set
- One subsystem corresponds to generic case, all others to singular of lower dimension
- Every solution of any subsystem gives a linearizing transformation and image of equation

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Examples

$$IsLinearizable := \left(\frac{d}{dx}y(x)\right)^{12} x + \left(\frac{d}{dx}y(x)\right)^{11} - \left(\frac{d}{dx}y(x)\right)^{9}\left(\frac{d^2}{dx^2}y(x)\right) + 105\left(\frac{d}{dx}y(x)\right)^{3}\left(\frac{d^2}{dx^2}y(x)\right)^{4} - 105\left(\frac{d}{dx}y(x)\right)^{4}\left(\frac{d^3}{dx^3}y(x)\right)\left(\frac{d^2}{dx^2}y(x)\right)^{2}$$
$$+ 15\left(\frac{d}{dx}y(x)\right)^{5}\left(\frac{d^4}{dx^4}y(x)\right)\left(\frac{d^2}{dx^2}y(x)\right) + 10\left(\frac{d}{dx}y(x)\right)^{5}\left(\frac{d^3}{dx^3}y(x)\right)^{2} - \left(\frac{d}{dx}y(x)\right)^{6}\left(\frac{d^5}{dx^5}y(x)\right)$$

Differential Thomas decomposition outputs a system of equations

$$\frac{\partial^2}{\partial x^2} f(x,y)$$
$$-\left(\frac{\partial^2}{\partial y\,\partial x} f(x,y)\right)$$
$$x\left(\frac{\partial}{\partial x} f(x,y)\right) - f(x,y) - \left(\frac{\partial}{\partial y} f(x,y)\right) - \left(\frac{\partial^2}{\partial y^2} f(x,y)\right) - \left(\frac{\partial^5}{\partial y^5} f(x,y)\right)$$
$$\frac{\partial}{\partial x} g(x,y)$$
$$A1(x,y)\left(\frac{\partial}{\partial y} g(x,y)\right) - A2(x,y)$$
$$-A2(x,y)^3 + A0(x,y)\,A1(x,y)$$
$$-A2(x,y)^4 + A1(x,y)^3$$
$$\frac{\partial}{\partial x} A2(x,y)$$
$$\frac{\partial}{\partial y} A2(x,y)$$

Computational time - 10s.

Examples

Classical Lie's formulas for second-order DE can also be obtained by using our algorithm, if we apply it to general form of candidate of linearization

$$y'' + F_3(x,y)(y')^3 + F_2(x,y)(y')^2 + F_1(x,y)y' + F(x,y) = 0.$$

Using appropriate block ranking of form $[[f,g],[F_1,F_2,F_3,F]]$, algorithm outputs

$$3\,F(x,y)\left(\frac{\partial}{\partial y}F2(x,y)\right) - 6\,F(x,y)\left(\frac{\partial}{\partial x}F3(x,y)\right) + 3\left(\frac{\partial}{\partial y}F(x,y)\right)F2(x,y) - 3\left(\frac{\partial}{\partial x}F(x,y)\right)F3(x,y) - 2\,F1(x,y)\left(\frac{\partial}{\partial y}F1(x,y)\right) + F1(x,y)\left(\frac{\partial}{\partial x}F2(x,y)\right) + 3\left(\frac{\partial^2}{\partial y^2}F(x,y)\right) - 2\left(\frac{\partial^2}{\partial y\,\partial x}F1(x,y)\right) + \frac{\partial^2}{\partial x^2}F2(x,y)$$

$$3\,F(x,y)\left(\frac{\partial}{\partial y}F3(x,y)\right) + 6\left(\frac{\partial}{\partial y}F(x,y)\right)F3(x,y) - 3\,F1(x,y)\left(\frac{\partial}{\partial x}F3(x,y)\right) - \left(\frac{\partial}{\partial y}F1(x,y)\right)F2(x,y) - 3\left(\frac{\partial}{\partial x}F1(x,y)\right)F3(x,y) + 2\,F2(x,y)\left(\frac{\partial}{\partial x}F2(x,y)\right) + \frac{\partial^2}{\partial y^2}F1(x,y) - 2\left(\frac{\partial^2}{\partial y\,\partial x}F2(x,y)\right) + 3\left(\frac{\partial^2}{\partial x^2}F3(x,y)\right)$$

Examples

Remark

Apart from ODE with polynomial coefficients, the suggested algorithmic approach is also applicable to the cases when the coefficients include elementary functions and also special functions defined by algebraic differential equations.

Instead:

$$IsLinearizable := \frac{d^4}{dx^4} y(x) + \left(\frac{d^2}{dx^2} y(x)\right) y + \sin(x) \left(\frac{d^2}{dx^2} y(x)\right) + \left(\frac{d}{dx} y(x)\right)^2$$

We should write:

$$IsLinearizable := \frac{d^4}{dx^4} y(x) + \left(\frac{d^2}{dx^2} y(x)\right) y + H(x, y) \left(\frac{d^2}{dx^2} y(x)\right) + \left(\frac{d}{dx} y(x)\right)^2$$

$$eq1 := \frac{\partial}{\partial y} H(x, y)$$

$$eq2 := \frac{\partial^2}{\partial x^2} H(x, y) + H(x, y)$$

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Analysis of Lie algebra

A lot of non-linearizable ordinary differential equations can be dropped by analysing its Lie algebra without full previous procedure. Every linear differential equation of n-th order admits Lie symmetry group of form

$$U = u_0(t) + \sum_{i=1}^{n} C_i u^{(i)}(t) + e^{C}(u - u_0(t)), T = t$$

with infinitesimal generators

$$\{u^{(i)}(t)\frac{\partial}{\partial u}, i = 1..n\}, (u - u_0(t))\frac{\partial}{\partial u}$$

Corollary

Lie symmetry algebra of linearizable equation has dimension n+1 or more.

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Lie symmetry algebra of linearizable equation has abelian subalgebra of dimension n.

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For $n > 2$ scalar differential equation is linearizable, if and only if its symmetry algebra contains abelian subalgebra of dimension n.

Sketch of proof (for $n = 3$):

- By transformation of variables one of operator could be reduced to the form $\frac{\partial}{\partial u}$
- Thus all components depend only on t
- Commutation relations imply that all operators are of form $u_i(t)\frac{\partial}{\partial u}$
- These operators generate group $U = u + \sum_{i=1}^{3} C_i u^{(i)}(t)$
- Only linear equations admit such group

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Abstract Lie symmetry algebra could be found explicitly without integration of determining equations (Reid'1991).

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Conjecture (by Boris Doubrov)

For $n > 2$ scalar differential equation is linearizable, if and only if
A) Lie symmetry algebra has dimension $n + 4$
or
B) Lie symmetry algebra has dimension $n + 2$ or $n + 1$, and its derived algebra is abelian and has dimension n.

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Conclusions

- We proposed an algorithmic approach to linearization of scalar ordinary differential equations, based on Lie procedure and analysis of symmetry algebra.
- Test-linearization is purely algorithmic, however to find linearizing transformation you have to find at least one solution of system of determining equations.
- Usually on practice this system consists of one-term and two-term equations and is easily solvable by Maple built-in routines
- In contrast to other results we do not make any restrictions on the order of differential equation
- In future we want to extend this result to system of second-order differential equations

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