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# The Research Seminar on Computer Algebra in 2009–2010

**S. A. Abramov[a], A. A. Bogolyubskaya[b], V. F. Edneral[c], and V. A. Rostovtsev[b]**

[a] *Dorodnicyn Computing Center, Russian Academy of Sciences, ul. Vavilova 40, Moscow, 119991 Russia*
[b] *Joint Institute for Nuclear Research, Dubna, Moscow oblast, 141980 Russia*
[c] *Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, 119992 Russia*
*e-mail: sergeyabramov@mail.ru; abogol@jinr.ru; edneral@theory.sinp.msu.ru; rost@jinr.ru*

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**Abstract**—An annual report on meetings of the scientific research seminar on computer algebra is presented.

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## 1. ABOUT THE SEMINAR

The subjects of the seminar are new results in computer algebra; more precisely, these are symbolic algorithms, their implementation, and related topics in system programming.

In the academic year of 2009–2010, the meetings were usually held monthly (every third Wednesday) at the Department of Computational Mathematics and Cybernetics of Moscow State University and at the Skobeltsyn Institute of Nuclear Physics. Like in the previous years, the meeting in May was organized in cooperation with the Laboratory of Information Technologies of the Joint Institute for Nuclear Research (JINR) and was held in Dubna at JINR. The Web site of the seminar at the address http://theory.sinp.msu.ru/dokuwiki//calg:main contains information about the scheduled and already presented talks.

## 2. REGULAR MEETINGS

Below is the list of presentations from September through April.[1]

S.P. Polyakov (Computing Center, Russian Academy of Sciences; s.p.polyakov@gmail.com). *Summation-related Symbolic Algorithms.*

Two approaches to summation of rational functions are studied. One approach avoids complete factorization of denominators, whereas the other approach is based on such factorization. In the framework of both approaches, algorithms of summation of rational functions are discussed, and new summation algorithms with additional degree minimization of the denominator of the summed part and the numerator of the remainder are proposed. Correctness of the Zeilenberger algorithm in the homogeneous case is proved. This case is shown to occur for any order of the telescoping operator. An algorithm for constructing a minimum-order annihilating operator is proposed. All algorithms discussed are implemented in Maple.

A.I. Zobnin (Department of Mathematics and Mechanics, Moscow State University; al_zobnin@mail.ru). *Ordinary Differential Polynomials and Differential Standard Bases.*

Solved and unsolved problems of the theory of differential standard bases (in particular, finiteness criteria with respect to various orderings) are considered.

Yu.A. Klimov (Keldysh Institute for Applied Mathematics, Russian Academy of Sciences; yuklimov@keldysh.ru). *Specializing Programs in Object-oriented Languages by Means of Partial Evaluation.*

Based on the existing partial evaluation methods for functional and object-oriented programs, a new method for specializing programs in object-oriented languages is proposed. Unlike the earlier partial evaluation techniques, the new method is capable of specializing all basic constructs of such languages as C# and Java and possesses a higher degree of polyvariance. The method has been implemented in the experimental specializer CILPE for object-oriented language CIL, which is an intermediate language of the Microsoft .NET platform, and tested on model programs. The speed-up was shown to be more than ten times.

A.V. Klimov (Keldysh Institute for Applied Mathematics, Russian Academy of Sciences; arkady.klimov@gmail.com). *Selection Tree Algebra and its Use in Algorithm Graph Builder and FORTRAN to Dataflow Language Compiler.*

Parallelizing compilation of FORTRAN requires analysis of the program for building its data flow graph (grid graph, algorithm graph). A method for building the algorithm graph based on the use of the selection trees is proposed. The selection tree is a special syntac-

[1] The list of presentations in the years of 1995–2009 can be found in [1–15].

tic structure convenient for representing both intermediate results of the analysis (such as statement effects and descriptions of states) and the graph itself. A set of operations on the selection trees is proposed in terms of which algorithms of program analysis and construction of the dependence graph are conveniently expressed. The proposed analysis is used for compiling FORTRAN onto the dataflow language PolyDFL.

A.A. Kytmanov (Siberian Federal University, Krasnoyarsk; kytmanov@lan.krasu.ru). *Construction of Computer Algebra Algorithms Based on the Function Theory Methods.*

An algorithm of elimination of unknowns from systems of nonlinear non-algebraic equations based on the multidimensional logarithmic residue and an algorithm for constructing a class of integral representations in certain domains of multidimensional complex space and a class of residues—analogues of the multidimensional logarithmic residue—are proposed.

A. Gheffar (XLIM, Université de Limoges, CNRS; f_gheffar@yahoo.fr) and S.A. Abramov (Computing Center, Russian Academy of Sciences, and Moscow State University; sergeyabramov@mail.ru). *Valuations of Rational Solutions of Linear Difference Equations at Irreducible Polynomials.*

Two algorithms are discussed that, for a given linear difference equation with rational function coefficients over a field $k$ of characteristic 0, construct a finite set $M$ of polynomials irreducible in $k[x]$ such that, if the given equation has a solution $F(x) \in k(x)$ and $\mathrm{val}_{p(x)}F(x) < 0$ for an irreducible $p(x)$, then $p(x) \in M$. After this, both algorithms compute a lower bound for $\mathrm{val}_{p(x)}F(x)$. The algorithms are applicable both to scalar linear equations and to linear systems of first-order equations. The algorithms are based on combinations of renewed approaches used in earlier algorithms for finding a universal denominator (Abramov and Barkatou) and denominator bounds (van Hoeij). A complexity analysis of the two proposed algorithms is presented.

A.A. Gusev, S.I. Vinitsky, O. Chuluunbaatar, and V.A. Rostovtsev (JINR, Dubna, and International University "Dubna"; gooseff@jinr.ru, vinitsky@thsun1.jinr.ru, chuka@jinr.ru, rost@jinr.ru). *Development of Symbolic–Numerical Algorithms for Solving the Low-Dimensional Boundary-Value Problems of Quantum Mechanics by the Kantorovich Method, by Reducing to Ordinary Differential Equations.*

Symbolic–numerical algorithms for solving elliptic boundary-value problems for impurity states in models of quantum dots, wires, and wells by the Kantorovich method are discussed. The algorithm is implemented in MAPLE–FORTRAN. In the future, it is possible to transfer to the Mathematica–Delphi platform.

M.G. Kokotchikova, D.S. Kulyabov, and L.A. Sevastianov (People's Friendship University, Moscow; mgkokotchikova@gmail.com, yamadharma@gmail.com, leonid.sevast@gmail.com). *A Regularized Method for Recovering a Function from Noisy Values of Its Partial Derivatives on a Grid.*

A problem of recovering a function defined on a circle $Q$ from its partial derivatives measured with errors on a grid of the Hartmann diaphragm $T = t_1, t_2, \ldots, t_k \in Q$ is solved. For stable recovering, the Tikhonov regularization method is used. On the basis of this method, a symbolic–numerical method for constructing a matrix of stabilizing functional was developed. The method was implemented in the computer algebra systems Axiom and Matlab.

## 3. TWO-DAY CONFERENCE AT THE JOINT INSTITUTE FOR NUCLEAR RESEARCH (DUBNA)

As has become a tradition, a joint meeting of the seminars of the Department of Computational Mathematics and Cybernetics of Moscow State University, of the Skobeltsyn Institute of Nuclear Physics, Moscow State University, and of the Laboratory of Information Technologies (LIT) of the Joint Institute for Nuclear Research was held in Dubna in May 2010. Essentially, this was a two-day conference on computer algebra and its applications.

The conference included the following presentations.

V.P. Gerdt (JINR, Dubna; gerdt@jinr.ru) and D. Robertz (RWTH Aachen, Germany; daniel@momo.math.rwth-aachen.de). *Algorithmic Consistency Check of Discrete Difference Approximations for Systems of Linear Partial Differential Equations.* An algorithm for checking consistency of finite difference approximations for systems of linear partial differential equations on orthogonal and uniform grids is suggested. It is based on construction and analysis of the Gröbner or involutive bases corresponding to the approximations.

S.A. Abramov (Computing Center, Russian Academy of Sciences, and Moscow State University; sergeyabramov@mail.ru). *On Some Decidable and Undecidable Problems Related to q-Difference Equations with Parameters.*

Linear $q$-difference equations with polynomial coefficients depending on parameters are considered. For the case where the ground field is $\mathbb{Q}(q)$, an algorithm is suggested that recognizes whether there exist numerical values of the parameters for which the equation has a polynomial (rational) solution. It is proved that such an algorithm does not exist if the parameters are allowed to be polynomials or rational functions of $q$.

A.M. Raportirenko (JINR, Dubna; ram@sunct1.jinr.ru). *PSL Version of REDUCE Running under 64-bit Windows.*

A realization of PSL (Portable Standard Lisp) and REDUCE for 64-bit family of operation system Win-

dows is discussed. The main drawback of all previous 32-bit realizations was memory limitation (not more than 128M). The present realization has no such limitation.

A.A. Gusev, S.I. Vinitsky, V.P. Gerdt, V.A. Rostovtsev, and O. Chuluunbaatar (JINR, Dubna, and International University "Dubna"; gooseff@jinr.ru, vinitsky@thsun1.jinr.ru, gerdt@jinr.ru, rost@jinr.ru, chuka@jinr.ru) T.A. Tolstova (Tver State University; tana0731@mail.ru). *A Symbolic–Numerical Algorithm for Reduction of Two-Dimensional Boundary-Value Problem by Using Parametric Functions.*

A symbolic–numerical algorithm for reduction of the two-dimensional elliptic boundary-value problem to a system of ordinary second-order differential equations is presented. The solution is expanded in terms of a set of functions of one variable, which depend on the second variable as a parameter. The algorithm is implemented in MAPLE–FORTRAN and is used for constructing asymptotic expansions of the considered boundary-value problem.

M.G. Kokotchikova, D.S. Kulyabov, and L.A. Sevastianov (People's Friendship University, Moscow; mgkokotchikova@gmail.com, yamadharma@gmail.com, leonid.sevast@gmail.com). *Application of the Regularized Method for Function Recovery in Optics* (continuation of the previous paper of the same authors (see Section 2)).

By virtue of the optical character of the considered problem, it is suggested to use the Zernike polynomials for function recovery. These polynomials form an orthonormal basis in the Hilbert space $L_2(Q)$. For regularized recovery of the matrix of discrete transform of the Hartmanngram to vectors of the Fourier coefficients of the optical surface, a symbolic–numerical method for constructing a matrix of stabilizing functional is used. The latter method is implemented in the computer algebra systems Axiom and Matlab.

N.M. Glazunov (National Aviation University, Kiev, Ukraine; glanm@yahoo.com). *Properties of Real Functions and Homological Algebra.*

Properties (behavior, singularities, existence, and extrema) of sufficiently smooth real functions are represented by interval categories and functors and studied by computer homological algebra methods. The method is shown to be applicable to studying nondifferentiable functions.

M.N. Gevorkyan and D.S. Kulyabov (People's Friendship University, Moscow; mngevorkyan@gmail.com, yamadharma@gmail.com). *An Operational Approach in Quantum Phase Problem.*

The problem of finding an analogue of the classical phase in the case of a quantum oscillator has not been completely solved yet. There are many approaches to solving this problem. In the paper, only those approaches are considered where the phase is interpreted as an operator. A survey of basic methods for constructing the phase operator is given. An implementation of the operators in the computer algebra system Maxima is described.

A.V. Korolkova and A.I. Tchernoivanov (People's Friendship University, Moscow; avkorolkova@gmail.com, tchernoivanov@gmail.com). *A Mathematical Model of Dynamic Data Transfer System.*

A model of a dynamic data transfer system based on the fluid model is proposed. The process that controls traffic flow conditions is defined by a random early detection (RED) algorithm. To analyze stability of solutions of the dynamic system and study types of self-oscillations, the computer algebra system Maxima is used.

A.V. Demidova and D.S. Kulyabov (People's Friendship University, Moscow; avdemid@gmail.com, yamadharma@gmail.com). *Derivation of Stochastic Differential Equations for Multidimensional Birth–Death Systems.*

Methods for obtaining stochastic differential equations (in particular, the Fokker–Plank equations) for systems described by multidimensional birth–death processes are considered. Implementations of these methods in the computer algebra system Maxima are proposed.

N.A. Nemchaninova and N.V. Lyubinskaya (People's Friendship University, Moscow; aryatasilva@gmail.com, nadin@sci.pfu.edu.ru). *Mathematical Modeling of Integrated Optical Waveguide in Curvilinear Coordinates.*

Modeling of behavior of electromagnetic waves in a waveguide with a Luneberg lens based on the use of curvilinear coordinates and a module in the Cadabra systems for automated transition from Cartesian to curvilinear coordinates are discussed.

V.V. Kornyak (JINR, Dubna; kornyak@jinr.ru). *Quantum Description of Finite Dynamical Systems.*

Specific features of gauge symmetries and quantum description in the framework of constructive finite dynamical systems are considered. The constructivity makes it possible to efficiently apply methods of computer algebra and computational group theory to studying quantum models of different types.

N.M. Glazunov and A.N. Timoshenko (National Aviation University, Kiev, Ukraine; glanm@yahoo.com, Dr.Aleksandr.Timoshenko@yandex.ru). *Computer Algebraic Aspects of Nondifferentiable Optimization.*

Let $E^n$ be an $n$-dimensional Euclidean space, $X$ be a closed convex set, and $f_i(x)$ be convex functions defined on some open set $Y \subseteq E^n$ that includes $X$, $i = 0, 1, \ldots, m$. The following problem of nondifferentiable optimization in the form of a convex programming problem is considered: $f^* = \inf f_0(x)$, $x \in X \subseteq E^n$, $f_i(x) \leq 0$. Computer algebra aspects of this problem, as well as of some other problems of nondifferentiable optimization, are studied.

D. Stefanescu (Bucharest University; stef@rms.unibuc.ro). *Construction of Some Irreducible Polynomials.*

A number of methods for constructing irreducible polynomials in one or two variables with integer coefficients are discussed. The main results are based on the factorization of Schönemann polynomials and study of Newton polygons related to bivariate polynomials. In turn, for checking irreducibility of difference polynomials, several methods using properties of slopes in the Newton polygon are suggested. Computational aspects of the theory are discussed.

Yu.A. Blinkov (Saratov State University, BlinkovUA@info.sgu.ru), V.P. Gerdt, (JINR, Dubna; gerdt@jinr.ru), and M.V. Zinin (Akronis LLC, Moscow; mzimin@gmail.com). *Comparative Analysis of Software Implementing Construction of Boolean Involutive Bases.*

On the basis of Boolean satisfiability (SAT), a comparative analysis of different implementations of algorithms for constructing Boolean involutive bases in C/C++ is given. The considered implementations use both standard and original memory managers, various representations of monomials and polynomials, Janet and Pommaret involutive divisions, and dense and sparse Janet trees for searching involutive divisors.

D.A. Yanovich (JINR, Dubna; yan@jinr.ru). *MPI-Aware Parallel Computation of Gröbner and Janet Bases.*

An MPI-aware parallel modification of the algorithm for computation of Gröbner and Janet bases is considered. Special attention is paid to minimizing exchanges between computational nodes. Benchmarks are presented.

A.A. Myllari and N.D. Gogin (The University of Turku, Finland; amyllari@gmail.com, alemio@utu.fi). *Weight-Spectra Polynomials for the Expected Waiting Time in the Penney Ante Game with a q-Sided Die.*

The paper is published in this issue of the journal.

M.A. Rybalkin (St. Petersburg State University; michael.rybalkin@gmail.com) and N.N. Vasiliev (St. Petersburg Division of the Steklov Institute of Mathematics, Russian Academy of Sciences; vasiliev@pdmi.ras.ru). *Permutation Binomials over Finite Fields and Rings Z/nZ.*

Properties of permutation binomials over finite fields and rings $Z/nZ$ are discussed. Generation of permutation binomials for arbitrarily large finite fields is studied. Possibility of generalizing the RSA cryptographic protocol with the use of permutation binomials is demonstrated.

A.B. Aranson (Scientific–Research Institute of Long-Range Radio Communication, Moscow; aranson@cbgrad.ru). *Calculation of Power Expansion Solutions of N. Kowalewski Modified ODE System by Power Geometry Algorithms.*

The paper is published in this issue of the journal.

I.V. Amirkhanov, D.Z. Muzafarov, N.R. Sarkar, I. Sarkhadov, and Z.A. Sharipov (JINR, Dubna; camir@jinr.ru, muzafarov@jinr.ru, nsarker@mail.ru, ibrohim@jinr.ru, ZARIF@jinr.ru). *Investigation of Solutions of Boundary Value Problems for the Quasipotential Equation Using the Shift Operation.*

Solutions of boundary value problems for the quasipotential equation with the Coulomb potential are studied with the use of the Maple system.

A.A. Bogolubskaya and I.L. Bogolubsky (JINR, Dubna; abogol@jinr.ru, bogolubs@jinr.ru). *On 2D solitons in SU(2) Gluodynamics.*

Possibility of solution existence in the two-dimensional $SU(2)$ gluodynamics is noted. The corresponding Hamiltonian obtained as a result of symbolic transformations in the Maple system is represented in terms of radial functions. Localized field distributions that provide local minima to this Hamiltonian are studied. Physical implications of such solutions are discussed.

N.M. Glazunov and V.S. Matsyuk (National Aviation University, Kiev, Ukraine; glanm@yahoo.com, Dr.Aleksandr.Timoshenko@yandex.ru). *On Computation of Tensor Fields on Manifolds.*

Methods for representing and computing the Riemannian, Ricci, and energy–momentum tensors on Riemannian and pseudo-Riemannian manifolds are considered. A method of computation of tensor products on tangent and cotangent bundles is presented.

I. Dimovski and M. Spiridonova (IMI, Bulgarian Academy of Sciences; dimovski@math.bas.bg, mspirid@math.bas.bg). *Resonances in Vibrating Systems with Integral Boundary Conditions.*

The paper is published in this issue of the journal.

S.I. Serdyukova and Yu.M. Shukrinov (JINR, Dubna; sis@jinr.ru, shukrinv@theor.jinr.ru). *A Numerical–Analytical Method of Calculating IVC for Josephson Junction Stack.*

The use of the Reduce 3.8 system for finding asymptotic terms in calculation of voltage in each point of IVC is discussed.

S.N. Tychkov (Institute for Control Sciences, Russian Academy of Sciences, Moscow; jevastiq@gmail.com). *On Harmonic Integrals.*

The paper addresses the question of what kind of vector field

$$\nabla = a(x, y)\partial_x + b(x, y)\partial_y$$

should be in order that solutions of the equation $\nabla u = 0$ be harmonic functions. The answer is that function $b(x, y)$ must satisfy the condition $\Delta b(b^2 + 1) - 2b(b_x^2 + b_y^2) = 0$.

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