Desingularization of leading matrices of systems of linear ordinary differential equations with polynomial coefficients

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We consider systems of linear ordinary differential equations containing m unknown functions of a single variable x. Coefficients of the systems are polynomials over a number field. Each of the systems consists of m independent equations. The equations are of arbitrary orders. We propose an algorithm which, given a system S of this type, constructs a nonzero polynomial $d(x)$ such that if S possesses an analytic solution having a singularity at α then the equality $d(\alpha) = 0$ is satisfied.

Linear differential equations (scalar or system) with variable coefficients appear in many areas of mathematics. Solving systems leads however to specific difficulties which do not appear in the scalar case. Consider the equation

$$P_r(x)y^{(r)} + P_{r-1}(x)y^{(r-1)} + \cdots + P_0(x)y = 0. \qquad (1)$$

First suppose that this is a scalar equation. The coefficients $P_0(x), P_1(x), \ldots, P_r(x)$ are polynomials, and $P_r(x)$ is not identically zero. If a solution of (1) has a singularity at some point α then $P_r(\alpha) = 0$.

If (1) is instead a system, $y = (y_1, y_2, \ldots, y_m)^T$ is a column vector of unknown functions of x and $P_0(x), P_1(x), \ldots, P_r(x)$ are square $m \times m$ matrices with polynomial entries then the role, which is played by the roots of the polynomial coefficient of $y^{(r)}(x)$ in the scalar case, can now be played by the roots of the determinants of the leading matrix $P_r(x)$, provided that this determinant is not identically zero. We study in this work the situation when $\det P_r(x)$ is the zero polynomial. Given a system S of the form (1), our algorithm Singsys finds a system S' of the same form (and with the same unknown functions), such that the determinant $d(x)$ of the leading matrix of S' is a nonzero polynomial, and the solutions space of the system S is a subspace of the solutions space of S'. The polynomial $d(x)$ is the result of the proposed algorithm execution. We have implemented the algorithm using the computer algebra system Maple ([4]).

Our approach can be used not only in the case of polynomial entries of matrices P_i in (1). However in other cases the equation $d(x) = 0$ may have an infinite set of roots.

The algorithm Singsys uses some of basic ideas of the algorithms EG and EG$'$ ([1, 2]) which are applicable to recurrence systems. The details of Singsys are to be presented in [3].

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References

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[3] Abramov S. A., Khmelnov D. E., *Singularities of solutions of linear differential systems with polynomial coefficients*, Journal of Mathematical Sciences (submitted).

[4] Maple online help: http://www.maplesoft.com/support/help/