Partial closed-form solutions of linear functional systems

Consider linear functional systems of the form $\theta Y = BY$ where B is a known matrix of coefficients, Y an unknown vector of functions and θ an operator such as differentiation or (q-)difference. Depending on the operator and the coefficient domain, there are several known algorithms for constructing the solutions of such systems in various classes of functions, such as polynomial, rational, hyperexponential or Liouvillian functions. But those algorithms only find solutions Y whose components are *all* in the specified class.

We address the following related problem: given a subset $\{Y_{e_1}, \ldots, Y_{e_m}\}$ of the entries of Y and an appropriate (i.e., closed under the action of skew-polynomials in θ) class of functions, find all solutions Y whose speficied entries are in the given class (more precisely we are interested in computing those entries only). For example, given a differential system $Y' = BY$, find all the rational functions that are Y_1 and Y_2-coordinates of some solution Y.

We present an algorithm that produces either one of two possible results:

- a proof that if the specified entries are in the given class, then all the other entries must be in that class too. Or,
- a new system involving the specified entries only (and some of their "derivatives"), and whose solution space is exactly the projection on those entries of the solutions of the initial system.

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