On the width of full rank linear differential systems with power series coefficients

S. A. Abramov, D. E. Khmelnov [1]

Computing Centre of the Russian Academy of Science,
Vavilova str., 40,
Moscow, 119333, Russia
E-mail: sergeyabramov@mail.ru, dennis_khmelnov@mail.ru

M. A. Barkatou

Institute XLIM, Université de Limoges, CNRS,
123, Av. A. Thomas, 87060 Limoges cedex, France
E-mail: moulay.barkatou@unilim.fr

We consider the following problem: given a linear ordinary differential system of arbitrary order with formal power series coefficients, decide whether the system has non-zero Laurent series solutions, and find all such solutions if they exist (in a truncated form preserving the space dimension). If the series coefficients of the original systems are represented algorithmically (thus we are not able, in general, to recognize whether a given series is equal to zero or not) then these problems are algorithmically undecidable ([2]). However, it turns out that they are decidable in the case when we know in advance that a given system is of full rank. Our proof is based in part on [1, 3, 4].

We prove additionally that the width of a given full rank system S with formal power series coefficients can be found algorithmically, where the width of S is the smallest non-negative integer w such that any l-truncation of S with $l \geq w$ is a full rank system. An example of a full rank system S and a non-negative integer l such that l-truncation of S is of full rank while its $(l+1)$-truncation is not, is given in the paper; however it is shown as well that the mentioned value w exists for any full rank system.

We propose corresponding algorithms and their Maple implementation, and report some experiments.

References

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