On the width of full rank linear differential systems with power series coefficients

S. A. Abramov, D. E. Khmelnov

Computing Centre of the Russian Academy of Science, Vavilova str., 40, Moscow, 119333, Russia
E-mail: sergeyabramov@mail.ru, dennis_khmelnov@mail.ru

M. A. Barkatou

Institute XLIM, Université de Limoges, CNRS, 123, Av. A. Thomas, 87060 Limoges cedex, France
E-mail: moulay.barkatou@unilim.fr

We consider the following problem: given a linear ordinary differential system of arbitrary order with formal power series coefficients, decide whether the system has non-zero Laurent series solutions, and find all such solutions if they exist (in a truncated form preserving the space dimension). If the series coefficients of the original systems are represented algorithmically (thus we are not able, in general, to recognize whether a given series is equal to zero or not) then these problems are algorithmically undecidable ([2]). However, it turns out that they are decidable in the case when we know in advance that a given system is of full rank. Our proof is based in part on [1, 3, 4]. We prove additionally that the width of a given full rank system $S$ with formal power series coefficients can be found algorithmically, where the width of $S$ is the smallest non-negative integer $w$ such that any $l$-truncation of $S$ with $l \geq w$ is a full rank system. An example of a full rank system $S$ and a non-negative integer $l$ such that $l$-truncation of $S$ is of full rank while its $(l+1)$-truncation is not, is given in the paper; however it is shown as well that the mentioned value $w$ exists for any full rank system.

We propose corresponding algorithms and their Maple implementation, and report some experiments.

References


1Work partially supported by the Russian Foundation for Basic Research, project no. 13-01-00182-a.