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The Fifth Conference “Computer Algebra” in Moscow

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Foreword

The fifth International Conference “Computer algebra”

`http://www.ccas.ru/ca/conference`

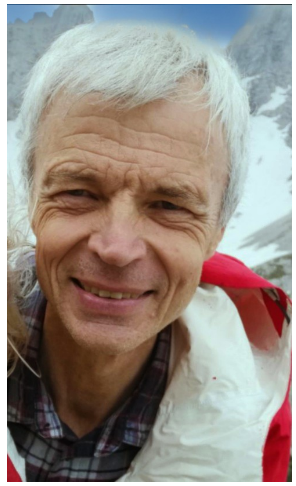
is organized in Moscow from 26 to 28 June 2023 jointly by the Dorodnicyn Computing Centre (Federal Research Center “Computer Science and Control”) of Russian Academy of Science, the Russian University of Peoples’ Friendship named after Patrice Lumumba and Keldysh Institute of Applied Mathematics of Russian Academy of Sciences.

The first, second, third and fourth conferences were held in Moscow in 2016, 2017, 2019 and 2021:

`http://www.ccas.ru/ca/conference2016`,
`http://www.ccas.ru/ca/conference2017`,
`http://www.ccas.ru/ca/conference2019`,
`http://www.ccas.ru/ca/conference2021`.

Computer algebra algorithms are focused on the exact solutions of mathematical and applied problems using a computer. The participants of this conference present new results obtained in this field.

During the Conference a special session in memory of Marko Petkovšek is held.



Marko Petkovšek
9.4.1955 – 24.3.2023

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1 Invited Talk Abstracts

Asymptotic Nonlinear Analysis as a Calculus and Applications

A.D. Bruno (Keldysh Institute of Applied Mathematics of RAS, Miusskaya sq. 4, Moscow, 125047, Russia)

In the last 60 years, there was formed an universal asymptotic nonlinear analysis, whose unified methods allow to find asymptotic forms and asymptotic expansions to solutions of nonlinear equations and systems of different types: algebraic, ordinary differential (ODE), partial differential (PDE) and systems of mixed-type equations. This are in two methods: (1) Reducing equations to the normal form and (2) Separating truncated equations. Two kinds of transformations of coordinate can be used to simplify the obtained equations: (A) Power and (B) Logarithmic.

In this lecture, the basic ideas of this calculus are explained for the simplest cases: a single algebraic equation in Section 1, Section 2 considers the autonomous ODE system. A single partial differential equation is considered in Section 3. An overview of applications are given in Section 4. Enlarged review was published in our recent work.

Hermite Reduction for D-finite Functions

Shaoshi Chen (1. KLMM, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; 2. School of Mathematical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China),
Lixin Du (Institute for Algebra, Johannes Kepler University, Linz, A4040, Austria),
Manuel Kauers (Institute for Algebra, Johannes Kepler University, Linz, A4040, Austria)

Trager's Hermite reduction solves the integration problem for algebraic functions via integral bases. A generalization of this algorithm to D-finite functions has so far been limited to the Fuchsian case. In the present paper, we remove this restriction and propose a reduction algorithm based on integral bases that is applicable to arbitrary D-finite functions.

A Saga on a Generating Function of the Squares of Legendre Polynomials

M. van Hoeij (Florida State University, Department of Mathematics, USA)

We decompose the generating function $\sum_{n=0}^{\infty} \binom{2n}{n} P_n(y)^2 z^n$ of the squares of Legendre polynomials as a product of periods of hyperelliptic curves. These periods satisfy *second* order differential equations which is highly unusual since *four* is the expected order for genus 2. These second order equations are arithmetic and yet their monodromy group is dense in $\mathrm{SL}_2(\mathbb{R})$. This implies that they cannot be solved in terms of hypergeometric functions, which is novel for an arithmetic equation that occurred naturally. This is joint work with Duco van Straten and Wadim Zudilin.

Two Methods for Efficient Generic Inversion

S. M. Watt (Cheriton School of Computer Science, University of Waterloo, Canada)

Two generic methods to compute multiplicative inverses are presented. These methods apply to integers, polynomials and matrices and are asymptotically faster than classical algorithms. The first method is to

use a modified Newton iteration to compute quotients via shifted inverses in a not-necessarily commutative Euclidean domain. The second method is to use the Moore-Penrose inverse to avoid pivots and whole-row operations in block matrix inversion.

Modular Arithmetic with Special Choice of Moduli

E.V. Zima (Wilfrid Laurier University, Waterloo, Canada)

Several methods of selection of moduli in modular arithmetic are discussed. With the proposed choice of moduli both modular reduction of an integer and reconstruction from modular images are accelerated. Special attention is paid to the moduli of the forms $2^n \pm 1$ and $2^n \pm 2^k \pm 1$. Different schemes of choice of these types of moduli and accelerated conversion of arbitrary precision integers into the modular representation and back are considered. Results of experimental implementation of different modular schemes confirm practicality of proposed methods.

2 Contributed Talk Abstracts

On Incomplete Rank Matrices

S.A. Abramov (Federal Research Center "Computer Science and Control" of RAS, Russia),
M. Petkovšek[1] (University of Ljubljana, Faculty of Mathematics and Physics, Slovenia),
A.A. Ryabenko (Federal Research Center "Computer Science and Control" of RAS, Russia)

Consider a matrix A of size $m \times n$ over a field K with $r = \mathrm{rank} A$ and $d = \min\{m, n\} - r > 0$, which implies that the rank of A is not full. We demonstrate that in such cases, it is possible to choose d elements from A such that, upon replacement of their values with other values from K, yield a matrix $\tilde{A}$ of full rank (when $m = n$, $\tilde{A}$ is nonsingular). We discuss as well the implications of this result for matrices with truncated formal series as their elements.

Power Algebra for Power Geometry

A.B. Aranson (Scientific Research Institute of Long-Range Radio Communication, Russia)

We suggest effective computation procedures for calculations by power geometry algorithms connected with Newton polyhedrons. These polyhedrons are suitable for visual explanations and graphical computations by hand in small dimension cases. But computer implementation of Newton polyhedron algorithms directly by A.D. Bruno definitions is too complicated and convoluted. Instead of geometrical definitions we use power substitution, linear inequalities and method similar to Cayley trick. We describe our methods in detail on example of calculating Puiseus expansions of solutions for Lotka-Volterra equations and then apply our methods for Euler-Poisson equations.

On Computation of Power Transformations

A.A. Azimov (Department of Algebra and Geometry, Samarkand State University after Sh. Rashidov, Uzbekistan),
A.D. Bruno (Keldysh Institute of Applied Mathematics of RAS, Miusskaya sq. 4, Moscow, 125047, Russia)

[1] Our friend and co-author Marko Petkovšek passed away on March 24, 2023 (*S. Abramov, A. Ryabenko).*

An algorithm for solving the following problem is described. Let $m < n$ integer vectors in the n-dimensional real space be given. Their linear span forms a linear subspace L in $\mathbb{R}^n$. It is required to find a unimodular matrix such that the linear transformation defined by it takes the subspace L into a coordinate subspace. Computer programs implementing the proposed algorithms and the power transforms for which they are designed are described.

Structure of Resonant Variety in Hamiltonian Systems with Three Degrees of Freedom

A.B. Batkhin (1. Keldysh Institute of Applied Mathematics of RAS, Miusskaya sq. 4, Moscow, 125047, Russia; 2. Department of Theoretical Mechanics, Moscow Institute of Physics and Technology, Russia),
Z.Kh. Khaydarov (Department of Algebra Geometry, Samarkand State University named after Sh. Rashidov, Uzbekistan)

For elementary singular point of a multiparameter Hamiltonian system we discuss a method of computing the condition of existence of a resonance of arbitrary order and multiplicity. For a certain resonant vector this condition defines a resonant variety as a variety in the space of coefficients of the characteristic polynomial of the linear part of the Hamiltonian system. By means of computer algebra and power geometry techniques polynomial parametrization of the resonant variety is proven. The obtained results can be used to investigate the formal stability regions of the equilibrium of a Hamiltonian multiparameter system as well as for the asymptotic integration of its normal form.

The First Differential Approximation on the Example of the Van der Pol Oscillator

Y. A. Blinkov (Saratov State University, Russia)

Systems of ordinary differential equations depending on parameters is considered, using the Van der Pol oscillator as an example. Advantages of the first differential approximation method and its implementation in the computer algebra systems are discussed. It is shown that, the presented method allows to estimate the stiffness of the Van der Pol oscillator and error of numerical methods and to propose simple criteria for choosing a step in calculations. The presented implementation of the method use a standard tools of computer algebra and can be applied systems with a polynomial right-had side.

Hermite Interpolation Polynomials on Parallelepipeds and FEM Applications

G. Chuluunbaatar (1. Joint Institute for Nuclear Research, Dubna, Russia; 2. Peoples' Friendship University of Russia, Russia),
A.A. Gusev (Joint Institute for Nuclear Research, Dubna, Russia),
O. Chuluunbaatar (Joint Institute for Nuclear Research, Dubna, Russia),
S.I. Vinitsky (1. Joint Institute for Nuclear Research, Dubna, Russia; 2. Peoples' Friendship University of Russia, Russia)

An algorithm for the analytical construction of multidimensional Hermite interpolation polynomials in a multidimensional hypercube is presented. In the case of a d-dimensional cube, the basis functions are determined by the products of d Hermite interpolation polynomials depending on each of the d variables given explicitly in analytic form. The efficiency of finite element schemes, algorithms and programs implemented in the MAPLE system is demonstrated by reference calculations of the Harmonic oscillator problem.

Asymptotic Approximations and Symbolic Representation of Parametric Families of Feedback Controls in Nonlinear Systems

Yu.E. Danik (Federal Research Center "Computer Science and Control" of RAS, Russia)
M.G. Dmitriev (Federal Research Center "Computer Science and Control" of RAS, Russia)

In this report the possibility of obtaining a symbolic description of a parametric family of synthesizing controls in nonlinear systems in the classical formulation, without constraints on the values of control functions, is considered based on the use of the Padé approximation technique and an approach to the synthesis of controls for nonlinear systems using the formal application of the Kalman-Letov algorithm.

Modeling of One-Step Processes Using Computer Algebra Tools

A.V. Demidova (Peoples' Friendship University of Russia, Russia),
O.V. Druzhinina (Federal Research Center "Computer Science and Control" of Russian Academy of Sciences, Russia),
O.N. Masina (Bunin Yelets State University, Russia),
A.A. Petrov (Bunin Yelets State University, Russia)

The issues of using computer algebra tools for modeling of dynamic systems whose behavior can be described by one-step processes are considered. An approach based on the representation of interactions between the elements of the system under study in the form of a graph is developed. This approach makes it possible, as a result of transformations, to obtain a symbolic representation of the differential equations of the model in both the stochastic and deterministic cases. The results can be used in solving problems of constructing and researching models of natural science.

Symbolic-Numerical Investigation of Asymptotic Method for Studying Waveguide Propagation Problems

D.V. Divakov (1. Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; 2. Joint Institute for Nuclear Research, 6 Joliot-Curie St, Dubna, Moscow Region, 141980, Russian Federation),
A.A. Tiutiunnik (1. Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; 2. Joint Institute for Nuclear Research, 6 Joliot-Curie St, Dubna, Moscow Region, 141980, Russian Federation)

A symbolic-numerical algorithm for solving the problem of waveguiding propagation of polarized light in irregular waveguides is considered. Within the framework of the adiabatic waveguide modes (AWM) model, the system of Maxwell's equations is reduced to a system of four ordinary differential equations and two algebraic equations for six components of the electromagnetic field in the zeroth approximation and the same number of equations in the first approximation. The paper describes a procedure for the symbolic reduction of Maxwell's equations to systems in the zeroth and first approximations of the AWM model. The steps of the symbolic-numerical method for solving the waveguide problem are described.

Integrable Cases of the Resonant Bautin System

V.F. Edneral (Skobeltsyn Institute of Nuclear Physics of Lomonosov Moscow State University, Russia)

Using the example of a polynomial resonance case of the Bautin system with parameters, we have written out the conditions for local integrability near stationary points and found restrictions on the parameters under which these conditions are satisfied. The resulting constraint is written as a system of algebraic equations for the ODE parameters. It is shown that for parameter values that are solutions of such an algebraic system, the ODE turns out to be integrable.

In this way we have found several cases of integrability. We propose a heuristic method that allows one to a priory determine the cases of integrability of an autonomous ODE with a polynomial right-hand side. The paper has an experimental character.

Analytical Geometry of the Projective Space $\mathbb{R}\mathrm{P}^3$ in Terms of Plücker Coordinates and Geometric Algebra

M.N. Gevorkyan (Peoples' Friendship University of Russia, Russia),
A.V. Korolkova (Peoples' Friendship University of Russia, Russia),
D.S. Kulyabov (1. Peoples' Friendship University of Russia, Russia; 2. Laboratory of Information Technologies Joint Institute for Nuclear Research 6 Joliot-Curie, Dubna, Moscow region, 141980, Russia)

Computer graphics uses a model of projective space to display three-dimensional scenes. The paper presents the basics of the analytic projective geometry of the space $\mathbb{R}\mathrm{P}^3$ in terms of Plücker coordinates. The interpretation of projective space and Plücker coordinates in terms of geometric algebra is also presented.

Generalized Power Series Solutions of q-Difference Equations and the Small Divisors Phenomenon

R.R. Gontsov (Institute for Information Transmission Problems of RAS, Russia),
I.V. Goryuchkina (Keldysh Institute of Applied Mathematics of RAS, Russia)

The problem of the convergence of generalized formal power series (with complex power exponents) solutions of q-difference equations is studied in the situation where the small divisors phenomenon arises; a sufficient condition of convergence generalizing corresponding conditions for classical power series solutions is obtained; an illustrating example is given.

Automatic Differentiation. Practical Aspects

A.Yu. Gorchakov (Federal Research Center "Computer Science and Control" of RAS, Russia),
V.I. Zubov (Federal Research Center "Computer Science and Control" of RAS, Russia)

The paper discusses the practical aspects of calculating derivatives in solving optimization and optimal control problems. Direct and inverse methods for calculating the gradient of scalar and vector functions are described. Methods of obtaining the gradient calculation code and packages implementing automatic differentiation methods are given.

An Optimized Procedure for Deciding Affinity of Finite Quasigroups

A.V. Galatenko (Lomonosov Moscow State University, Russia),
A.E. Pankratiev (Lomonosov Moscow State University, Russia),
R.A. Zhigliaev (Lomonosov Moscow State University, Russia)

Finite quasigroups are becoming a popular platform for the design of cryptographic primitives. Quasigroup affinity is one of the crucial properties in the framework of cryptography. In our paper we propose an optimized procedure that decides affinity with the complexity $O\left(n^2 \log n\right)$, where n is the quasigroup order, and present the results of experiments with the software implementation.

Symbolic Investigation of the Plane Equilibria of the System of Two Connected Bodies on a Circular Orbit

S.A. Gutnik (1. MGIMO University, Russia; 2. Moscow Institute of Physics and Technology, Russia)

Computer algebra methods are used to study the plane equilibrium orientations of a system of two bodies connected by a spherical hinge that moves on a circular orbit. The main attention is given to the study of the equilibria of the two-body system in the plane of the circular orbit, in the plane perpendicular to the circular orbital plane and in the plane tangent to the circular orbital plane. A new method is proposed for transforming the system of trigonometric equations determining the equilibria into a system of polynomial equations, which in turn are reduced by calculating the resultant to a single algebraic equation. The domains with an identical number of equilibria are classified using algebraic methods for constructing a discriminant hypersurface.

Regularity Criterion for a Linear Differential System with Meromorphic Coefficients

D.O. Ilyukhin (Volgograd MOU secondary school No. 18, Russia),
A.V. Parusnikova (HSE University, Russia)

Regularity criterion for the singular point of a linear meromorphic system is obtained. Its proof is based on trasform of the system to a linear equation with meromorphic coefficients. We can check, whether the singular point of the system is regular using computer algebra system.

Symbolic Calculus for Optimal Control in Multi-Agent Economic Model

B.B. Iusup-Akhunov (The Phystech School of Applied Mathematics and Informatics, Moscow Institute of Physics and Technology, Russia),
I.G. Kamenev (Federal Research Center "Computer Science and Control" of RAS, Russia),
A.A. Zhukova (Federal Research Center "Computer Science and Control" of RAS, Russia),
N.P. Pilnik (1. Federal Research Center "Computer Science and Control" of RAS, Russia; 2. Department of Applied Economics, Faculty of Economic Sciences, HSE University, Russia)

This paper presents the development of the support system for modeling socio-economic processes based on an open source platform. This system is based on the approach of the ECOMOD system developed for complex economic models where agents plan decisions based on optimal control problems. The system enables analysis of a model containing a set of several agents, each either solving an optimal control planning problem or following a scenario. The combined descriptions form a complex system of nonlinear relations, which is difficult to write down without errors in mathematical expressions on paper or using computer. The system includes elements for verifying the correctness of the model record: balances, dimensions etc.

Algorithm EG as a Tool for Finding Laurent Solutions of Linear Differential Systems with Truncated Series Coefficients

D.E. Khmelnov (Federal Research Center "Computer Science and Control" of RAS, Russia),
A.A. Ryabenko (Federal Research Center "Computer Science and Control" of RAS, Russia)

Laurent solutions of systems of linear ordinary differential equations with the truncated power series coefficients are considered. The Laurent series in the solutions are also truncated. We use induced recurrent systems for constructing the solutions and have previously proposed an algorithm for the case when the induced system has a non-singular leading matrix. The algorithm finds the maximum possible number of terms of the series in the solutions that are invariant with respect to any prolongation of the original system. Below we present advances in extending our algorithm to the case when the leading matrix is singular using algorithm EG as an auxiliary tool.

On the States of N-Level Quantum System With Positive Wigner Function

A. Khvedelidze (1. Institute of Quantum Physics and Engineering Technologies, Georgian Technical University, Tbilisi, Georgia; 2. Andrea Razmadze Mathematical Institute of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia; 3. Meshcheryakov Laboratory of Information Technologies, Joint Institute for Nuclear Research, Dubna, Russia),
A. Torosyan (Meshcheryakov Laboratory of Information Technologies, Joint Institute for Nuclear Research, Dubna, Russia)

In the present report, within the phase-space formulation of quantum theory of N-level systems, we discuss the existence of "classical states" which are defined as those states whose Wigner function is positive semi-definite. An explicit description of a set of classical states is given using the associated convex bodies inside the simplex of density matrices' eigenvalues. It is demonstrated how these results allow one to calculate three measures of classicality constructed out of the quasiprobability distributions: the nonclassicality distance, Kenfack-Życzkowski indicator, and the global indicator.

A Constructive Approach to Problems of Quantum Mechanics

V.V. Kornyak (Laboratory of Information Technologies, Joint Institute for Nuclear Research, Dubna, Russia)

We consider a constructive modification of quantum mechanics based on permutation representations of finite groups in Hilbert spaces over cyclotomic fields, and its connection with the Weyl–Schwinger "finite quantum mechanics". Constructive quantum mechanics requires mathematical tools that differ significantly from those used in traditional continuous theory: number theory, finite fields, complex Hadamard matrices, finite geometries, etc. A natural approach to the various problems that arise in the field are computer calculations based on the methods of computer algebra and computational group theory.

Nonlinear Effects of Motion Near the Equilibrium Manifold of Nonholonomic Systems

A.S. Kuleshov (Department of Mechanics and Mathematics, Lomonosov Moscow State University, Russia),
N.M. Vidov (Department of Mechanics and Mathematics, Lomonosov Moscow State University, Russia)

A general analysis of nonlinear oscillations of conservative nonholonomic systems, possessing the equilibrium manifold is presented. The procedure of normalization of equations of motion near the equilibrium manifold is discussed. As an example of the general theory the problem of motion of a heavy rigid thin rod on a perfectly rough right circular cylinder is considered.

Primitive Elements of Free Non-Associative Algebras over Finite Fields

M.V. Maisuradze (Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Russia), A.A. Mikhalev (Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Russia)

The representation of elements of free non-associative algebras in the form of a set of multidimensional tables of coefficients is determined. The operation of finding partial derivatives of elements of free non-associative algebras in the same form is considered. Using this representation, a criterion for the primitiveness of elements of length two in terms of matrix ranks is obtained, as well as a primitivity test of elements of arbitrary length. With this test, the number of primitive elements with two generators was estimated.

Computation of Tropical Sequences Associated with Somos Sequences in Gfan Package

F. Mikhailov (Saint Petersburg Electrotechnical University "LETI", Russia)

The main objective of this work is to study tropical recurrent sequences associated with Somos sequences. For a set of tropical recurrent sequences, D. Grigoriev put forward a hypothesis of stabilization of the maximum dimensions of solutions to systems of tropical equations given by polynomials, which depend on the length of the sequence under consideration. The validity of such a hypothesis would make it possible to calculate the dimensions of these solutions for systems of arbitrary length. The main purpose of this work is to compute tropical sequences associated with Somos sequences using the Gfan package and to test the Grigoriev hypothesis.

Bounding the Support in the Differential Elimination Problem

Y.S. Mukhina (Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Russia)

We will discuss bounding the support in the differential elimination problem. The main result is the characterisation of the support of the result of the differential elimination for a planar system with generic polynomials of fixed degree in the right-hand side.

On Tight and Efficient Bound Propagation For Neural Networks Based on Bernstein Polynomial Approximations

M. Wu (School of Computer Science and Software Engineering, East China Normal University, Shanghai 200062, China)

Bound propagation is a critical step in wide range of Neural Network model checkers and reachability analysis tools. So far, linear and convex optimizations have been used to perform bound propagation, however, these methods suffer from introducing large errors due to the high non-convexness. In this work, we study several techniques on how to produce both tight and efficient bound propagation for Neural Networks based on Bernstein polynomial approximations.

A Note on Application of Program Specialization to Computer Algebra

A.P. Nemytykh (Program Systems Institute of RAS, Russia)

Given an input program P, program specialization aims at run-time optimization of P w.r.t. its sintactic structures. The simplest example is to generate a definition of a (partial) subfunction of the (partial) function defined by P. One may wonder in some of syntactic properties of the program q resulted by specialization rather than run-time of q. We show a number of corollaries of known mathematical constructions, which derived by a general purpose tool, a specializer, and shortly introduce those properties of the tool, that allow it to achieve such interesting results. The idea of using such a tool for generating mathematical formulae was originated by Alexandr Korlukov.

Learning Port-Hamiltonian Systems

V.N. Salnikov (CNRS / La Rochelle University, France)

The port-Hamiltonian approach provides a natural formalism for studying mechanical and physical systems with interaction or dissipation. We address the problem of recovering an optimal port-Hamiltonian structure for systems of ordinary differential equations. The procedure includes a machine learning based phase – isolating the nodes in the connectivity graph of the system and a “deterministic” phase – spelling-out the internal geometric structure of each node.

On a Simple Lower Bound for the Matrix Rank

A.V. Seliverstov (Institute for Information Transmission Problems of Russian Academy of Sciences (Kharkevich Institute), Russia)

Over a field of characteristic not equal to two, we proved a lower bound for the rank of a square matrix, where every entry outside the leading diagonal is equal to either zero or one, but every diagonal entry is neither zero nor one. This lower bound equals half of the order of the matrix. It is tight.

Calculations of Quantum Corrections in Supersymmetric Theories Using Computer Algebra Methods

I.E. Shirokov (Department of Theoretical Physics, Faculty of Physics, Lomonosov Moscow State University, Russia)

We propose a symbolic algorithm and a C++ program for generating and calculating supersymmetric Feynman diagrams for $\mathcal{N} = 1$ supersymmetric electrodynamics. The program generates all diagrams that are necessary to calculate a specific contribution to the two-point Green function of matter superfields in the needed order, and then reduces the answer to the sum of Euclidean momentum integrals.

Optimisation of Computer Algebra Techniques Application for Rician Data Analysis

T.V. Yakovleva (Federal Research Center “Computer Science and Control” of RAS, Russia)

The paper presents a mathematical research directed on the optimization of the computer-algebra methods' application for solving the task of stochastic data analysis. Within the conducted theoretical investigation a few mathematical techniques of the statistical data analysis have been elaborated which allow essential simplifying of solving the task by computer algebra methods. The developed two-parameter approach to data analysis is efficiently applicable to a wide spectrum of scientific and applied tasks, in which the signal to be analyzed is described by the Rice statistical model.