Methods of reduced directions for sequential and parallel computers

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Let us consider nonlinear programming problem

$$\min\{f_0(x), x \in \Omega\}, \Omega = \{x \in E^n : f_j(x) \le 0\},$$

$j \in J = \{\overline{1,m}\}$ The functions supposed to be sufficiently smooth. The iterative process

$$x_{k+1} = x_k + t_k\, s_k,\ k = 0, 1, ...$$

is defined to solve this problem. Here s_k is the direction of moving from current iterative point x_k, t_k – the step along this direction.

The methods for solving nonlinear programming problem have been constructed in the uniform scheme of methods of reduced direction for sequential computers [1]. This approach is based on the idea of linearization of "active" constraints and the notion of "reduced" direction used as search direction in the iterative points. Different cost functions have been used to calculate the step length along "reduced" direction. The methods of following known groups have been realized into the frame of the scheme mentioned above : the feasible directions methods [2], the nondifferentiable cost functions [3], the differentiable [4] and barrier cost functions [5], the modified Lagrangian functions [6], methods of centers [5], combined methods [7,8].

To build "reduced" direction the several strategies of constructing the set of "active" constraints gradients are used. The idea of linearization of active constraints in the current iterative point x is used: $f_j(x) + \langle f_j'(x), s\rangle = -v_j,\ j \in J_a$,

$v = (v_j)_{\,j\in J_a}$ — the vector of parameters. Direction has been defined by the following formula:

$$s = -Pz - R(v + f).$$

Here vectors z, v — are the direction parameters, matrices P, R satisfy the following conditions:

$$A^T P = 0,\ A^T R = I_r,\ A = (f_j'(x))_{\,j\in J_a},$$

where I_r is an $r \times r$ unit matrix, $r = J_a \le n$. There are different ways to calculate matrices P, R. We implement the following technique of matrices definition, based on the LQ matrix decomposition:

$$P = Q_2^T,\ R = Q_1^T L^{-1},\ A^T = (L; 0)Q,$$

where L — is left thriangular matrix, Q — orthogonal matrix.

Hence to determine the new iterative point x_{k+1} it is necessary to define the cases of calculating the direction parameters z, v and step length t. Parameters and step length are caculated so that to decrease some merit function. For this reason the following merit functions are used:

$f_0(x)$ — target function [2]

$F_N(x) = f_0(x) + N \max\limits_{j\in J}\{0, f_j(x)\},\ N > 0$ — exact cost function [3]

$\Phi_\rho(x) = f_0(x) + \dfrac{1}{2\rho}\sum\limits_{j\in J}\max{}^2\{0, f_j(x)\}$ — differentiable cost function [4]

$B_\rho(x) = f_0(x) + \rho\sum\limits_{j\in J}\ln(-f_j(x))$ — barrier cost function [5]

$M_\rho(x,\lambda) = f_0(x) + \dfrac{\rho}{2}\sum\limits_{j\in J}\{\max{}^2(0, \lambda_j + \dfrac{1}{\rho}f_j(x)) - \lambda_j^2\}$ — modified Lagrangian function [6]

$\Phi_\beta(x) = ((f_0(x) - \beta)^+)^2 + \sum\limits_{j\in J}\max{}^2\{0, f_j(x)\}.$ — external distance function [5]

On the basis of the uniform scheme of reduced directions methods the optimization system ODiS with visualization was realized for sequential computers. It is used for research purposes and also for studying of methods of nonlinear optimization.

It is suggested to use algorithm of barrier cost functions method to build parralel algorithms of methods of reduced directions. Three ways of parallelization in the uniform scheme are used:

- calculating algorithms of methods of reduced gradient calculate the value of constraints of the problem many times so they could be calculated parallely;
- the uniform scheme is based on matrix-vector multiplications, so it is possible to make their effective parallel realization;
- there are parallel realizations of LQ matrix decomposition used in the uniform scheme.

The direction is built parametrically in the methods iof reduced directions. Them method to use is fixed with merit function, the way of active constraints set calculation and with direction parameters. In that a way all the methods are contracted in the uniform scheme, the same basic procedures are used to realize all the methods with special parameters.

The procedures of direction and step calculation have been written in the MPI interface for methods of reduced gradient, The parametes to chose the method to use are availiable.

The experiments are conducting now on the example of barrier cost functions method [9] using written procedures. Results of numerical experiments of application of parallel algorithms of methods of the reduced directions are received.

References

[1] V. S. Izhutkin and M. V. Petropavlovskii, *Methods of Reduced Directions with Different Merit Functions for Nonlinear Programming Problem.* In Operations Research, A.Karmann, K.Mosler, M.Schader, G.Uebe, eds, Physica-Verlag, Heidelberg, pp.188-189, 1993.

[2] V. S. Izhutkin and M. Yu. Kokurin, *The Reduced Directions Methods with Feasible Points for Nonlinear Programming Problems.* USSR Computitional Mathematics and Mathematical Physics, Vol.30, N 1, pp.159-169, 1990.

[3] V. S. Izhutkin and M. Yu. Kokurin, *The Reduced Directions Methods for Nonlinear Programming Problems.* USSR Computitional Mathematics and Mathematical Physics, Vol.28, N 6, pp.135-144, 1988.

[4] V. S. Izhutkin and M. V. Petropavlovskii, *Methods of"Reduced" Directions, Based on Differentiable Penalty Function, for a Problem of Non-linear Programming.* Russian Mathematics (Iz VUZ), Vol.38, N 12, pp.50-59, 1994.

[5] V. S. Izhutkin and M. V. Petropavlovskii and A. V. Blinov, *Methods of Centers and Barrier Function with Using Reduced Direction for Problem of Nonlinear Programming.* Russian Mathematics (Iz VUZ), Vol. 40, N 12, 1996.

[6] V. S. Izhutkin and M. V. Petropavlovskii, *Adduced Direction Method Based on Modified Lagrange Function for Problem of Nonlinear Programming.* Russian Mathematics (Iz VUZ), Vol.39, N 12, pp.33-42, 1995.

[7] V. S. Izhutkin and O. V. Bastrakova, *Combine Method of Reduced Gradient for Nonlinear Programming Problem.* Abstracts of Symposium on Operations Reseach (SOR'99), Magdeburg, September 1-3, p.65, 1999.

[8] V. S. Izhutkin and O. V. Bastrakova, *Four-Stage Combine Reduced Direction Method for Nonlinear Programming Problem.* Abstracts of International Conference on Operations Research (OR'2000), Dresden, p.27, 2000.

[9] V. S. Izhutkin and O. V. Bastrakova, *Parallel Algorithm of Barrier Cost Function Method in the Uniform Scheme of Methods of Reduced Directions.* Abstracts of International Conference on Operations Research (OR'2000), Dresden, September 9-12, pp.26-27, 2000.