Qualitative study of the behavior of a system of ordinary differential equations in a distributed computing environment

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Let's consider a normal system of ordinary differential Equations

$$\dot{x} = f(x), \tag{1}$$

where $x = (x^1, ..., x^n)$ – vector function of a real variable t, and $f = (f^1, ..., f^n)$ – vector polynomial.

If all the solutions of system (1) are defined on whole axis R, then (1) defines a dynamical system g^t, such that every solution $x(t)$ of this system ssubject to the condition

$$g^t x(0) = x(t).$$

Qualitative research of system g^t almost certainly connected with the investigation of behavior of discrete dynamical system $g^{NT}, N = 0, 1, 2, \dots,$ where T – some positive number (see for example [1]). Nevertheless, if we restrict by the case of solutions of the system (1), contained in some compact set Σ, then by virtue of the autonomous system (1), it is enough to construct only the operator g^T. The last is possible in virtue of form of the right side of the system (1).

In fact, using the Picard method of successive approximations allows us to construct an approximation to the solution of (1) in view of

$$x_{i+1}(t) = x(0) + \int_0^t f(x_i(\tau))d\tau. \tag{2}$$

If to take

$$x_0(t) \equiv x(0),$$

then scheme (2) will converge uniformly to $x(t)$ on some sufficiently small interval $[0, T]$, that defines by the operator g^T. And modern grid computer technologies makes it easy to carry out all necessary calculations symbolically.

As an application of the results the solutions of the Lorenz system were investigated

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = ax - y - xz, \\ \dot{z} = xy - bz \end{cases} \tag{3}$$

with classical values of parameters: $\sigma = 10$, $a = 28$, $b = 8/3$. Calculations were performed up to the mark fortieth and trajectory of system (3) were constructed for a wide range of initial conditions. Particularly, Table 1 shows the results of calculation of trajectory described by the solution with initial condition

$$x_0 = -15.720831,\ y_0 = -16.587193,\ z_0 = 36.091132, \tag{4}$$

taken in the immediate vicinity of the attractor.

Table 1: The solution of system (3) with initial condition (4).

N	t	$x(t)$	$y(t)$	$z(t)$
1	0	-15.720831	-16.587193	36.091132
2	17.334	-15.659134	-16.566101	35.954224
3	45.017	-15.652555	-16.566140	35.938103
4	67.104	-15.689783	-16.635305	35.964610
5	86.686	-15.812414	-16.750982	36.164029

In calculation result there was not found any convincing signs of the existence of cycles in the Lorenz system (see table 1); to make absolutely sure about this it is enough, for example, to construct a vector field of system (3) in the present points. That is why, since there was not fixed any significant reduction in the speed of a point, it is more likely to assume that trajectory of the system – compact minimal set, that consists of non-closed recurrent trajectories (see for example [1]). This trajectories, probably are not even almost periodic because some local divergence of typical trajectories during the computation was observed (see for example [1]).

The analytical work that was conducted after has rigorously shown that hypothesis about the structure of the attractor in the Lorenz system is not correct: the given attractor does not contain a countable everywhere dense set of cycles. That is why there is no evidence to suggest that all cycles (if any exists) are saddle. The last fully confirmed by a high-precision numerical experiment.

References

[1] *Nemyckiy V.V., Stepanov V.V.* Kachestvennaya teoriya differencial'nyh uravneniy(Qualitative theory of differential equations). M.: URSS, 2004.