Application of global search algorithms to solve of optimal control problem in system of four semiconductor quantum dots

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Employment of quantum dots (QD) as basic elements of a quantum computer has a lot of advantages. We can control of their geometric characteristics and location. It is proposed to use vertically combined layers of QD in Ge-Si system for carrying out of quantum computation. We consider two cells of a quantum computer which are based on four tunnel-coupled semiconductor QD. The size and the density of QD is determined to existence of sufficient tunnel couple in top layer for implementation of quantum logical operations. The tunnel couple in lower layer is not exist. A voltage impulse applies to metal gates, which are above QD, lead to tunnelling of electrons from lower to top dots. The exchange operation of information (SWAP) implements due to movement particles to a top layer. The problem consists in finding optimal form and length of the voltage impulse for realization of SWAP operation.

Nonstationary Schrodinger eguation is employed for description of controlled electrons movement in system of vertically combined QD. The mathematical model of analyzed subject is formulated as bilinear controlled system of ordinary differential equations:

$$\dot{x} = A_0(u)x,$$

where matrix dimension $A_0(u)$ is 32×32 [1]. It is necessary to transfer the system from $x(t_0) = x^0$ to $x(t_1) = x^1$ with minimum of expended energy. The energy required for movement of electrons between different QD is a control. It is formulated the aim functional which contains two parts: a control variation in all time interval and a deviation of a system trajectory from terminal conditions

$$I(u) = s_1 \int_0^T u^2 dt + s_2 \sum_{j=1}^{32} (x_j(t_1) - x_j^1(t_1))^2 \to min.$$

We deal with variants of this problem with different values of coefficients s_1, s_2 and functional of response speed ($I(u) = t_1 \to min$).

Solutions of considered series problems implement by global algorithms realized in software OPTCON-I and OPTCON-III [2–4]. There are curvilinear search algorithm, tunneling algorithm, algorithm of sequantial sampling and others. However, we could not to solve proposed optimal control problem with using standard computing scheme as in many applied problems. It is required to develop a special computing technology recognized specifics of the optimization model.

The first trouble of numerical solving of this problem is a slow convergence of algorithms in all time interval. We use well-known method of parameter continuation for overcoming of this trouble, the system of differential equations is supplemented with a new parameter p: $\dot{\bar{x}} = p_1 f(x, u, t)$. A numerical experiment in small time interval allows us to get close approximation for carry out next computation. It is found the strategy of parameter continuation which permit to pass from solving of simple problems to initial one with accounting results from previous stages.

The most complicated and laborious problems turn out a response speed problem. A time estimation of response speed was given by experts and was equal 1500 units. We can make better this estimation and find an optimal value proved to be 606 conventional units [5].

The multistage series of numerical computations for minimization of aim functional and for investigation of system dependence from changes of model parameters values are carry out. It is found optimal form of controlled voltage impulse and trajectories of probability of electron's staying in different QD by developed global algorithms for solving optimal control problems. It is detect that an existence of a tunnel couple between QD in vertical line leads to appearance additional errors for implementation of SWAP operation for electrons in lower layer. The results of study of electron states in model with two vertically combined layers of tunnel-coupled semiconductor QD show a possibility of using these states for quantum computing.

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