Variational inequality with the connected restrictions for a inconsistent problem of mathematical programming

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Let's consider the following problem of mathematical programming:

$$\max_x \{f_0(x) \,|\, F_0(x) \leq 0,\, x \in X\}. \quad (1)$$

Here $X \subseteq R^n$, $f_0 : R^n \to R$, $F_0 : R^n \to R^m$.

The problem (1) is widely known and there is a set of methods of its solution. However in case of inconsistent initial model (for example, when the quantity of available resources does not allow to execute the chosen technological processes) system of restrictions will be incompatible, and corresponding optimization problem (1) becomes incompatible. One of the approaches offered by the academician I. I. Eremin for correction of such tasks, is the addition of parameters into an initial problem and definition of parameters ensuring solubility of problem. Thus it is possible to optimize expenses for correction of a problem [1].

For an incompatible optimization problem (1) we shall enter parameters $y^* \in R^m_+$, then the problem (1) receive the following form:

$$\max_x \{f(x, y^*) \,|\, F(x, y^*) \leq 0,\, x \in X\}. \quad (2)$$

Here $f : R^n \times R^m_+ \to R$, $F : R^n \times R^m_+ \to R^m$.

For the decision of the received parametrical problem (2) we shall enter complementarity problem:

$$\max_y \{(y, F(x^*, y)) \,|\, F(x^*, y) \leq 0,\; y \in R^m_+\}. \quad (3)$$

In such form it is possible to consider a pair of problems (2), (3) as a pair of mutual-inverse problems.

Let's write down the received pair of problems as a variational inequality with the connected restrictions. Let's enter for this purpose new target function and new function with adequate connected restrictions:

$$H(v, w) = f(x_2, y_1) - (y_2, F(x_1, y_2)),$$
$$G(v, w) = [F(x_2, y_1), F(x_1, y_2)],$$

where $v = [x_1, y_1]$, $w = [x_2, y_2]$, $v, w \in \Omega = X \times R^m_+$. In the terms of these functions we shall formulate a new problem:
to find a vector $v^* \in \Omega$ for the decision problem

$$\min_w \{H(v^*, w) \,|\, G(v^*, w) \leq 0,\; w \in \Omega\}. \quad (4)$$

The decision of a problem (4) will the decision of an initial problem (2), (3).

If the criterion function $H(v, w)$ differentiability on second variable, a problem (4) is possible to present as a variational inequality with the connected restrictions

$$(\nabla_w H(v^*, v^*), w - v^*) \geq 0,\; \forall\, w \in \Omega,\; G(v^*, w) \leq 0,$$

where $\nabla_w H(v, v) = \nabla_w H(v, w)|_{v=w}$.

Thus, have received model of an incompatible problem of mathematical programming as a variational inequality with the connected restrictions. The representation of an incompatible problem of mathematical programming in such form is new, and in particular, allows to apply a wider class of methods to the decision of similar problems.

References

[1] I. I. Eremin *Inconsistent models of optimum planning.* - M.: Science, 1988.