Methods of the efficiency assessing of investment projects in uncertainty conditions and risks of their implementation

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The efficiency of investment projects (IP) can be estimated on the basis of several criteria (NPV, IRR, PI and others). NPV is one, which represents the discounted amount on an annual basis the project net cash flow (Net Cash Flow). The higher value the project is better. The remaining criteria are derived from NPV.

In case of NPV many experts put forward well-founded criticism, which essentially boils down to the fact that this criterion is rather abstract measure. NPV reflects the mathematical goal of the project, due to the strong dependence of its value from the discount, NPV doesn't take into account other (social, environmental) aspects of the effectiveness of IP. NPV underestimates the effectiveness of IP with long-term implementation compared with the speculative stock investments, etc. However, despite the deficiencies noted a real alternative to isn't currently exist. In this context improving the quality of the IP effectiveness assessments may be due to deficiencies in the methodology of calculating the values of this criterion.

In our opinion, one of the major drawbacks NPV is incorrect assessment of the uncertainties of financial flows and the discount rate and risk during the period of IP.

In reality, risks and uncertainties reduce the value of the project NPV. Adequately assess the impact of these effects on the efficiency of IP is possible by taking them into account in terms of criteria, which can be presented in one of the following versions:

$$N\hat{P}V = \sum_{t=1}^{T} \frac{(\hat{I}_t - \hat{O}_t)}{(1+E_n)^t}, \qquad (1)$$

$$N\hat{P}V = \sum_{t=1}^{T} \frac{(\hat{I}_t - \hat{O}_t - \sum_{i=1}^{n}(\hat{R}_{it} - Z_{it}))}{(1+E_n)^t} \qquad (2)$$

where $\hat{I}_t$ is inflow of funds for the project in year t,

$\hat{O}_t$ is outflow of funds for the project in year t,

E_n is discount the project in year t,

All of these variables are considered as random.

$\hat{I}_t$, $\hat{O}_t$, E_n - deterministic values;

$\hat{R}_{it}$ - project effectiveness reducing risk in year t for the i-reason, estimated as a random variable, taking into account the cost of its reduction Z_{it}, which are deterministic values, i=1,n.

It is easy to show that the expression (1) and (2) are equivalent. For example, (1) can be expressed in project risk as downside risks to the tributaries of the values $\hat{I}_t$ and increasing outflows $\hat{O}_t$. In all cases, the NPV estimate should be regarded as a random variable. The nature and form of its distribution can be determined by taking into account the uncertainty of initial information, i.e. variables: $\hat{I}_t$, $\hat{O}_t$, E_n, $\hat{R}_{it}$.

In general, their uncertainty can be classified into three grades.

1. Statistical uncertainty (low), characterized by the known laws of distribution of the variables: $\hat{I}_t$, $\hat{O}_t$, E_n, $\hat{R}_{it}$.

2. The average degree of uncertainty, in which can be shaped membership function of the variables $\hat{I}_t$, $\hat{O}_t$, E_n, $\hat{R}_{it}$ to certain intervals of the existence of their values.

3. Interval uncertainty (high), in which can only be defined boundaries of the intervals of the existence of these variables.

The presentation of the criterion of NPV is uniquely determined by the presentation of its member variables. It can be shown that when we have the interval expression indices $\hat{I}_t$, $\hat{O}_t$, E_n, the criterion of NPV determined by the interval of its values [NPV1; NPV2], where the indices 1 and 2 respectively describe the lower and upper bounds of the interval of existence of the criterion, which are difficult to estimate based on the rules of "interval arithmetic". In this situation, the criterion for evaluating the NPV can take its value, estimated using, for example, the Hurwitz criterion:

$$\text{NPV*} = (1-\lambda)\text{NPV}_1 - \lambda\text{NPV}_2, \quad (3)$$

where λ is investors optimization parameter $(\lambda \approx 0{,}3)$.

In the conditions of an average degree of uncertainty based on membership functions of variables: $\hat{I}_t$, $\hat{O}_t$, E_n, using the rules of mathematics of fuzzy sets can be built and the membership function of the test. The most straightforward procedure for its construction and triangular and trapezoidal membership functions. In this case, the "cautious" attitude of investors to risk the estimated value NPV* can be defined as the left ordinate α-cut of its membership function, where α-level of this function, which expresses the degree of credibility of the investor.

Procedures for assessing the criteria for IP interval for the original forms of the information representation are quite detailed described in the scientific literature. However, NPV project assessment procedure with a statistical uncertainty of this information remains virtually undeveloped, apparently because of their complexity. This type of uncertainty characterized by the known laws (density functions) of the distribution parameters: $\hat{I}_t$, $\hat{O}_t$, E_n, $\hat{R}_{it}$. For example, for projects related to the development of oil and gas, they can be constructed on the basis of information on flows of projects carried out under similar conditions of occurrence and volume of raw materials, taking into account the forecasts of price volatility.

Theoretically, if we have certain, but different distributions laws of the indicators included in the NPV, it's possible to form the density function of its distribution by using the convolution of densities of functions of relevant indicators. However, this procedure is quite complicated. For example, one can show that for two random variables distributed according to the exponential and normal distribution, their sum has the density of the form:

$$f(z) = \lambda e^{-\lambda z} e^{\frac{\lambda^2\sigma^2 - \bar{x}\lambda}{2}} F\left(\frac{z - \bar{x} - \lambda\sigma^2}{\sigma}\right) \quad (4)$$

where $\lambda \sim N(\bar{\lambda};\sigma^2)$, $y \sim \lambda e^{-\lambda z}$, z =x+y are random variables with the corresponding density functions, $\bar{x}$ and λ are expectations of the random variables, F (u) – value of the standardized normal distribution at the point u.

Naturally, in the presence of dozens of differently variables included in the criterion of NPV, the convolution procedure is practically impossible to implement. Furthermore, if the discount and the project is a random variable, in this case, use the procedure for constructing the density function of random variable is the ratio of two quantities, which is characterized by greater density.

To solve this problem it is expedient to use some simplifying assumptions, according to which estimated only the average mean and variance of NPV and bases on the known estimates of the variables characteristics included in the NPV. NPV distribution law can be assumed normal, that is justified by many considered for the calculation of its variables.

Considering this assumption, the variance of NPV can be estimated on the basis of the following expressions:

$$D(N\hat{P}V) = \sum_t D\left(\frac{x_t}{(1+E_n)^t}\right) \quad (5)$$

where D is a symbol of the variance;

x_t is a random variable formed by the sum of the random flow of the project in year t.

Dispersion $D = \left(\frac{x_t}{(1+E_n)^t}\right)$ can be obtained using the Taylor expansion ratio of these random variables. For example, with this assumption that E_n=const we can show that the variance of NPV takes the following form:

$$D(N\hat{V}P) = \sum_t \frac{(M[u])^2 \cdot \sigma^2{}_{xt} + (M[x_t])^2 \cdot \sigma^2{}_{ut}}{(M[u_t]^4)} =$$

$$= \sum \frac{\sigma^2{}_{xt} + (M[\hat{x}_t])^2 \cdot t^2 (1+E_n)^{-2} \sigma^2 E_n}{(1+E_n)^{2t}} \qquad (6)$$

where $u = (1+\hat{E}_N)^t$,

$\sigma^2{}_u = t^2 (1+E_n)^{2(t-1)} \cdot \sigma_{En}$,

$M[u] = (1+E_n)^t$,

σ^2 is symbol of the variance of z, M [z] is symbol of its expectation.

Based on the assumption of a normal NPV distribution lawn, its estimated value can be estimated as the left NPV density function quintile that is formed by the known values of the average mean and variance of this indicator. Assessing the significance of quintiles is given confidence level p=p(NPV>NPV*), reflecting investor attitude towards risk.

In conclusion, we note that the content of the approaches to assessment criterion value NPV differs only in the forms of expression of uncertainty used in its calculation of the initial information.

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