Global optimization methods based on efficient partitioning strategies *

Ya. D. Sergeyev, D. E. Kvasov

University of Calabria, Italy
N. I. Lobachevsky University of Nizhni Novgorod, Russia

E-mails:
yaro@si.deis.unical.it
kvadim@hotmail.ru

In many practical applications of global optimization, the objective function $f(x)$ is defined over a hyperinterval $D \subset R^N$ and can be black-box, multiextremal, and requiring high computational resources to be evaluated (see, e.g., [1, 3, 8, 10, 17, 16, 18, 19]). Such problems often can not be solved by traditional optimization techniques making strong suppositions about the problem (convexity, differentiability, etc.).

Numerous iterative processes proposed in literature (see, e.g.,[1, 2, 6, 9, 10, 11, 14, 15, 18, 19]) for solving such problems can be distinguished depending on the way they combine the following features:

(i) the strategy used for partitioning the search region D;

(ii) the way to choose one or more elements for the next partition;

(iii) the number of evaluation points over the new subregions obtained after each partition;

(iv) the location of these points within each of the new subregions.

For example, one-point-based, diagonal and simplicial algorithms can be considered relatively to the properties (i) and (iii). One-point-based algorithms subsequently subdivide the search region in smaller ones and evaluate the objective function at one point within each subregion (see, e.g., [1, 2, 4, 12]). In the case of diagonal algorithms, the multidimensional domain is partitioned adaptively into a set of hyperintervals, and the objective function is evaluated only at the vertices corresponding to the main diagonal of each new hyperinterval (see, e.g., [7, 10, 11, 13, 14]). Simplicial algorithms partition the region in simplexes and evaluate the objective vector function at all their vertices (see, e.g., [3, 5, 19]).

In this communication, it will be shown that partition strategies themselves, independently of feature (ii), can influence significantly the number of computationally extensive function evaluations made by an algorithm. For example, in the framework of widely used diagonal algorithms, traditional partition strategies (like bisection or partition on 2^N subintervals; see, e.g., [3, 7, 10]) do not fulfill the requirements of computational efficiency because of the execution of many redundant evaluations of the objective function. A new diagonal adaptive partition strategy (see, e.g., [11, 12, 13, 14]) allowing one to avoid such a computational redundancy will be discussed. In contrast to the traditional diagonal partition strategies, the new one produces regular meshes of the function evaluation points and significantly outperforms the traditional strategies in terms of the number of function evaluations.

*This research was supported by the grants 64729.2010.9 and MK-3473.2010.1 awarded by the President of the Russian Federation for supporting the leading research groups and young researchers, respectively, as well as by the grant 11-01-00682-a awarded by the Russian Foundation for Fundamental Research.

It will be shown how this partitioning strategy can be used in both the frameworks of one-point-based algorithms and diagonal algorithms. Particularly, several examples of global optimization methods based on the efficient partitioning strategy will be presented, their convergence properties will be discussed, and results of numerical comparisons with some numerical methods (see, e.g., [3, 4, 14]) frequently used for solving real-life global optimization problems will be reported.

References

[1] Yu. G. Evtushenko. *Methods of Solving Extremal Problems and Their Application in Optimization Systems.* Moscow: Nauka, 1982. (In Russian).

[2] Yu. G. Evtushenko, M. A. Posypkin. Coverings for global optimization of partial-integer nonlinear problems. *Doklady Mathematics*, Vol. 83(2), pp. 1–4, 2011.

[3] C. A. Floudas, P. M. Pardalos, Eds. *Encyclopedia of Optimization* (6 Volumes). Kluwer Academic Publishers, 2001. (The 2nd edition: Springer, 2009).

[4] D. R. Jones, C. D. Perttunen, B. E. Stuckman. Lipschitzian optimization without the Lipschitz constant. *J. Optim. Theory Appl.*, Vol. 79(1), pp. 157–181, 1993.

[5] R. Horst, H. Tuy, *Global Optimization – Deterministic Approaches.* Berlin: Springer, 1996.

[6] R. Horst, P. M. Pardalos, Eds. *Handbook of Global Optimization*, Vol. 1. Dordrecht: Kluwer Academic Publishers, 1995.

[7] D. E. Kvasov, C. Pizzuti, Ya. D. Sergeyev. Local tuning and partition strategies for diagonal GO methods. *Numer. Math.*, Vol. 94(1), pp. 93–106, 2003.

[8] D. E. Kvasov, D. Menniti, A. Pinnarelli, Ya. D. Sergeyev, N. Sorrentino. Tuning fuzzy power-system stabilizers in multi-machine systems by global optimization algorithms based on efficient domain partitions. *Electr. Power Syst. Res.*, Vol. 78(7), pp. 1217–1229, 2008.

[9] D. Lera, Ya. D. Sergeyev. Lipschitz and Hölder global optimization using space-filling curves. *Appl. Numer. Math.*, Vol. 60(1-2), pp. 115–129, 2010.

[10] J. D. Pintér. *Global Optimization in Action.* Dordrecht: Kluwer Academic Publishers, 1996.

[11] Ya. D. Sergeyev. An efficient strategy for adaptive partition of N-dimensional intervals in the framework of diagonal algorithms. *J. Optim. Theory Appl.*, Vol. 107(1), pp. 145–168, 2000.

[12] Ya. D. Sergeyev. Efficient partition of N-dimensional intervals in the framework of one-point-based algorithms. *J. Optim. Theory Appl.*, Vol. 124(2), pp. 503–510, 2005.

[13] Ya. D. Sergeyev, D. E. Kvasov. Global search based on efficient diagonal partitions and a set of Lipschitz constants. *SIAM J. Optim.*, Vol. 16(3), pp. 910–937, 2006.

[14] Ya. D. Sergeyev, D. E. Kvasov. *Diagonal Global Optimization Methods.* Moscow: Fizmatlit, 2008. (In Russian).

[15] Ya. D. Sergeyev, D. E. Kvasov. Lipschitz global optimization. In: Cochran J. J. et al., Eds. *Encyclopedia of Operations Research and Management Science* (8 Volume Set), Vol. 4, pp. 2812–2828. New York: Wiley, 2011.

[16] Ya. D. Sergeyev, P. Daponte, D. Grimaldi, A. Molinaro. Two methods for solving optimization problems arising in electronic measurements and electrical engineering. *SIAM J. Optim.*, Vol. 10(1), pp. 1–21, 1999.

[17] A. S. Strekalovsky. *Elements of Nonconvex Optimization.* Novosibirsk: Nauka, 2003. (In Russian).

[18] R. G. Strongin, Ya. D. Sergeyev. *Global Optimization with Non-Convex Constraints: Sequential and Parallel Algorithms.* Dordrecht: Kluwer Academic Publishers, 2000.

[19] A. A. Zhigljavsky, A. Žilinskas. *Stochastic Global Optimization.* New York: Springer, 2008.