Control of dynamical regimes of systems with deterministic chaos.

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We consider essentially nonlinear dynamic systems in which the possible existence of deterministic chaos. This means that the solution of the system whose parameters are completely deterministic, can in a certain range of parameter values exhibit stochastic behavior. For other parameters the solution can be fully determined. Of particular interest are areas in the parameter space, where there is a transition from deterministic to chaotic behavior of the solution [1]. It is often implemented complex polyharmonic solutions, which are difficult to view as you approach the parameters of the field of chaos. In some systems, implemented a scenario of transition to chaos by period doubling when changing settings [2].

The problem of selecting individual parameters to control the behavior of the solution arises. At the same time may be set different goals. For example, a solution must be completely deterministic. Or a solution must be chaotic. Or a solution must meet the state transition from deterministic to chaotic.

As a simple example, consider a one-dimensional nonlinear dynamical system described by Duffing equation with negative stiffness [1].

$$\frac{d^2x(t)}{dt^2} - 10x + \frac{dx}{dt} + 100x^3 = W\cos 3.76t \quad (1)$$

As the only control parameter is selected amplitude of the external effects W. In this system, in a certain range of values of variable parameters may be implemented simplest periodic solutions (Fig. 1).

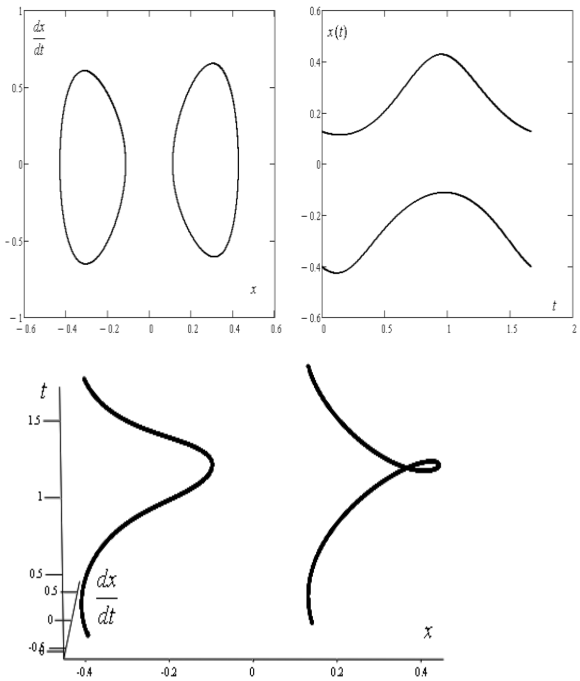

Fig. 1. Phase trajectories and one period of T-periodic stable solution of equation (1) W = 0.7.

In a certain range of values of the control parameter W solution is chaotic behavior (Fig. 2).

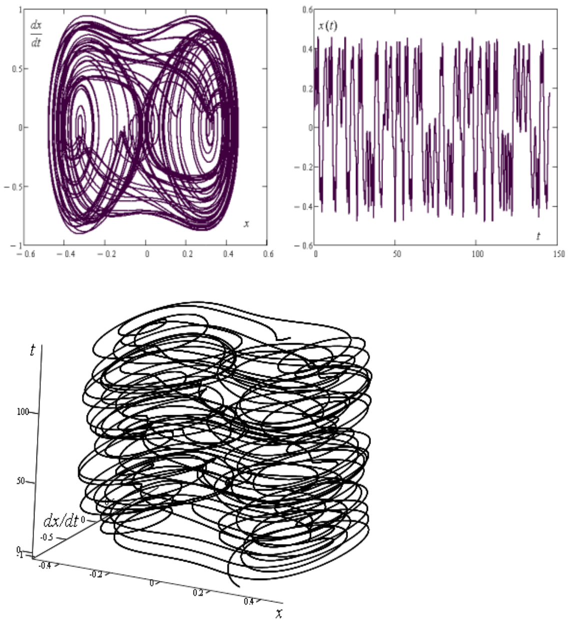

Fig. 2. Phase trajectories and chaotic solution of equation (1) W = 1.5.

On the boundary values of the parameters between regular and chaotic behavior of solutions of period-doubling bifurcations occur (Fig. 3)

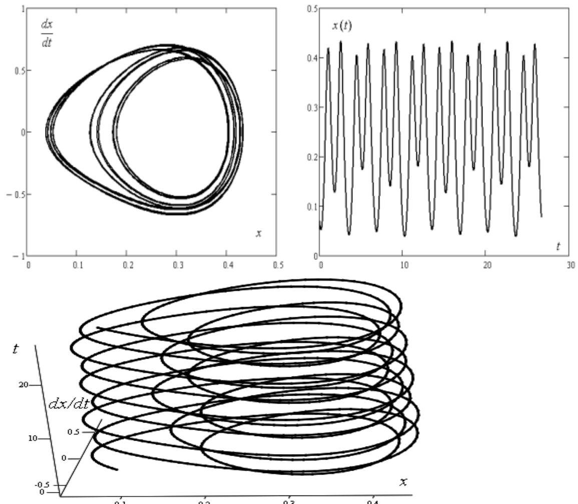

Fig 3. Phase trajectories and one period of 16T-periodic stable solution of equation (1) W = 0.9379

In addition, within the range of the parameter corresponding to the chaotic behavior of solutions, there is a narrow region of existence of complex stable periodic solutions with different periods, different from the period of the system (Fig. 4).

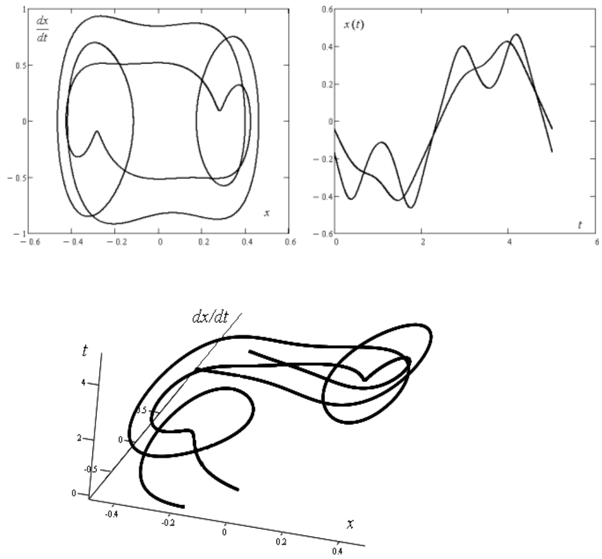

Fig. 4. Phase trajectories and one period of different stable 3T-periodic solutions of (1) W = 1.70.

In this simple example to control the chaotic behavior of the system used a single variable parameter W, which has the meaning of the amplitude of external influence. Another important parameter control the chaotic behavior of a generalized dissipation.

Generalization of the simple system (1) is a multi-dimensional nonlinear dynamic system (2), corresponding to the model of nonlinear oscillation systems with several degrees of freedom.

$$\frac{d^2x_j(t)}{dt^2}+j^2(k^2j^2+D_1)x_j(t)+x_j(t)\frac{j^2}{4}\left(\sum_{m=1}^{N}m^2x_m^2(t)\right)+$$
$$+\delta_j\frac{dx_j(t)}{dt}=Q_j(t),$$

$$j=1,2,\dots,N. \qquad (2)$$

In this system there are similar effects in more complex and diverse form, taking into account the mutual influence of dynamical regimes corresponding to different degrees of freedom. (Fig.5)

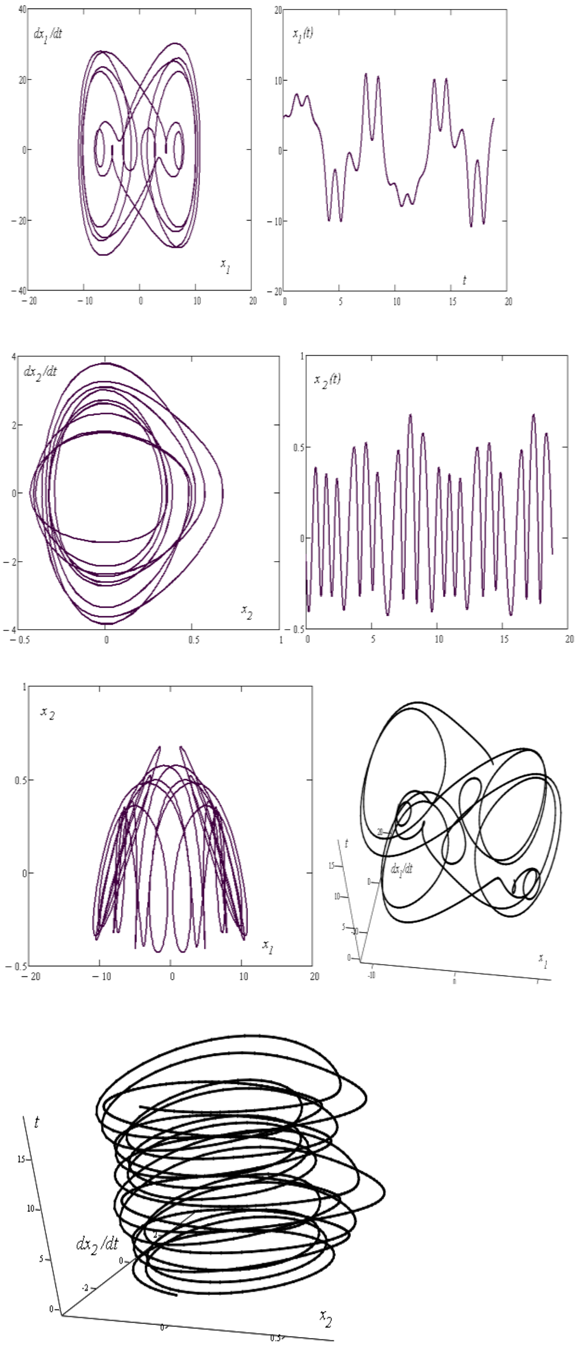

Fig. 5. Stable 3T-periodic solution of system (2) at k2 = 0.5, D1 = 0.5, δ1 = δ2 = 0.1, Q = 100, ω = 1.

The parameters control the qualitative and quantitative characteristics of the solutions here, you can also use the dissipative characteristics, parameters of external influence and the degree of mutual influence of dynamical regimes corresponding to different forms of vibration.

Generalizing this approach, similar tools control the states of models with dynamical chaos can be used in the study of the dynamics of more complex systems [3].

References

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[3] L.F. Petrov. Nonlinear effects in economic dynamic models.- Nonlinear Analysis 71 (2009) e2366□e2371.