# On Insurable Irreversible Investments under Ambiguity *

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Tools of financial mathematics have long been applied to theory of real options, especially irreversible investment. We introduce the concept of *insurable irreversible investment* which allows to treat certain real options as (perpetual) American straddles (i.e. a combination of a put and a call).

Straddles in finance are held by agents who expect market to be significantly volatile, but are not sure about the direction of the future market movement. By holding the call and put on the same asset they are effectively insured against big movements of market in any direction. Similarly, when the firm is facing an investment decision which can be, in a certain sense, insured it is in fact holder of both real call (investment opportunity) and real call (insurance). If the decision is the one that can 'make or break' the business depending on the market condition, the firm facing this insurable irreversible investment opportunity is insured against future market movements in the same sense that the holder of the straddle is insured against movements of the financial market. Our mathematical analysis allows for direct applications in cases of both real options and financial derivatives.

We work in the classical framework of Black-Scholes-Samuelson and use optimal stopping as our main tool for analysis. We consider two settings: first, the firm faces uncertainty modelled by risk, and, second, the firm faces uncertainty modelled by risk and ambiguity. When the agent is certain about the probability measures over the possible states of the world we say that the agent is facing the risk, and this is the classical meaning of risk. Ambiguity is the way to model the uncertainty when the agent is not sure or cannot identify the probability measure over the states of the world. The modern approach to ambiguity is inspired by [1] where agent considers several possible probability measures and behaves, in a certain sense, pessimistically. There are several applicable formalizations of this kind of ambiguity of which we use $\kappa$-ignorance, which effectively models the uncertainty about the drift of the underlying geometric Brownian motion.

In the first part we work in the classical non-ambiguous setting, i.e. a setting where the agent knows the probability measure over the states of the world. Classical connection between optimal stopping and free boundary problems is exploited and the problem is reduced to a well known non-linear system of equations. Arguably the main novelty of this work is reduction of the aforementioned system to one nonlinear equation. This is a useful novelty in the literature of both financial options and real options, and allows for easy application to pricing certain investment opportunities as well as classical straddles.

In the second part we consider the ambiguous setting. We briefly review the ambiguity model of $\kappa$-ignorance as introduced in [2]. Optimal stopping theory in this and more general settings has been studied in [3]. Since these results are quite recent, with the hope of presenting a relatively self contained work, we briefly consider their theory particularly adjusted/simplified for our case of $\kappa$-ignorance. There the main mathematical tools used are g-expectation and g-martingales developed by Shige Peng and several other authors and we show the basics of their functioning and applications.

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With all the necessary tools available for considering the optimal stopping problems under ambiguity we consider the insurable irreversible investment problem once again, but now under ambiguity. We provide the full analysis and comment on the similarities and differences with non-ambiguous case.

Since this work is in the classical setting of Black-Scholes-Samuelson and simple model of ambiguity ($\kappa$-ambiguity) we provide guidelines for future research in pricing of perpetual American straddles and insurable irreversible investments.

## References


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