Development of a Bee Colony Optimization Algorithm for the Capacitated Plant Location Problem

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Recently much attention is given to the approximate methods of solution of combinatorial optimization problems. Special interest is conserned with the algorithms which are constructed on analogies with biological and physical processes.

Such algorithms are heuristic and, as a rule, work well from the point of view of CPU time and quality of solutions obtained. The most known of them are genetic algorithms, algorithms of an ant colony, simulated annealing, variable neighborhood search (see, for example, [3, 9]). In the given work the algorithm of a bee colony [10, 12] is considered.

This heuristic, which has received active progress last years, models the behavior of melliferous bees during searching of sources of nectar.

In nature, bees having found the plants allocating nectar, return to a beehive carrying out the typical movements which have received the name *recruiting dances*. Dances and special signals of a bee report the information about a distance up to a source of forage and a direction of a flight to blossoming plants, involving the other bees of a colony into the search. Depending on intensity of a dance, the quantity of bees going to already known plants and to investigation of new melliferous herb is adjusted.

The algorithm of a bee colony (BC) for the first time has been offered by Lučić and Teodorović for a problem of the Travelling salesman problem in 2001 [10].

This method is developing actively; the new works have appeared. These works are devoted to use of BC for a vehicle routing problem (Lučić, Teodorović, 2003), the maximal satisfability problem (Drias, Sadeg, Yahi, 2005), scheduling problems (Chong, Low, Sivakumar, Gay, 2006), the p-median problem (Teodorović, Šelmić, 2007), the p-center problem (Davidović, Ranljak, Šelmić, 2010), for the simple plant location problem [7] and others (see, for example, [12, 13]).

In the given work a new version of algorithm of a bee colony for a well-known capacitated plant location problem [2, 4, 11] is offered and its experimental research is carried aut.

1 Problem Formulation

Let us consider the problem in the following statement. There are a set of potential locations for plants with fixed costs and capacities, and a set of customers with demands for goods supplied from these plants. The transportation costs from the plants to each customer are given. The problem is to find the subset of plants that will minimize the total fixed and transportation costs so that demand of all customers can be satisfied without violating the capacity constraints of the plants.

For construction of an integer linear programming model let us introduce the following notations:
I is the set of sites of possible plant locations, $i \in I = \{1,...,m\}$; J is the set of customers, $j \in J = \{1,\ldots,n\}$; $T = (t_{ij})$ is the matrix, where t_{ij} is a transportation cost for goods supplied from the plant i to the customer j, $i \in I, j \in J$; c_i is the cost of opening of a plant in site i (in other words plant i), $i \in I$; a_i is the capacity of the plant i,

$i \in I$; b_j is the demand of the customer j, $j \in J$.

We will use the variables:
$z_i = 1$, if the plant is opened in site i , and 0 otherwise; $X = (x_{ij})$ is a matrix of deliveries of a made product, x_{ij} is a quantity of product transported from a plant i to a customer j, here $i \in I, j \in J$.

Then the mathematical model of the problem will look like:

$$F(z, X) = \sum_{i \in I} c_i z_i + \sum_{i \in I} \sum_{j \in J} t_{ij} x_{ij} \to \min \tag{1}$$

subject to

$$\sum_{i \in I} x_{ij} = b_j,\ j \in J, \tag{2}$$

$$\sum_{j \in J} x_{ij} \le a_i z_i,\ i \in I, \tag{3}$$

$$x_{ij} \ge 0,\ i \in I,\ j \in J, \tag{4}$$

$$z_i \in \{0, 1\},\ i \in I. \tag{5}$$

Equalities (2) mean that the demand of each client should be satisfied. Conditions (3) consider volumes of manufacture and guarantee impossibility of service by the closed plants.

A special case of this problem is the Simple plant location problem which is NP-hard in a strong sense [5].

Note, that it is not difficult to define the elements of matrix X on the basis of z from (2)-(5), therefore we will name the vector z *a solution* to the problem.

2 Bee Colony Optimization Algorithm for the Capacitated Plant Location Problem

For a description of the algorithm we will draw an analogy between the behavior of bees and the process of a finding of the optimal solution of the discrete location problem. The set of admissible solutions of a problem corresponds to a field, where the bee looks for a source of nectar, and a point of admissible area corresponds to a flower that produces nectar. The quantity of nectar is estimated by a value of an objective function.

The basic algorithm of the bee colony Bee Colony Optimization (BCO) offered in [12], consists of two stages: "flight forward" and "back flight". The first of them corresponds to the behavior of the melliferous bee searching for the sources of nectar. At this stage the space of solutions is investigated and a new partial solution is constructed. The second stage simulates the behavior of a bee after its return to a beehive. During the "back flight", all bees, depending on the received result, are divided into two groups: "foragers", completing the record solution, and the "scouts" building a new one. Such way with two versions of constructing of a new solution is a distinctive feature of the bee colony algorithm.

In the given work this process is considered from the point of view of methods of local search. It represents heuristics with special rules of finding a solution. Differences from the basic algorithm are, for example, in construction of the new solution. For fulfilling this step in the algorithm BCO a bee tries to improve the available partial solution for a number of times, beginning either with the current record ("forager"), or from the new zero solution $z = \vec{0}$ ("scout"). In the proposed algorithm LBCO (Local search Bee Colony Optimization), the search for a new solution is carried out by two algorithms of an artificial bee using a neighborhood. The algorithm of a bee ϕ ("forager") chooses z from the set neighborhood of a vector z_{rec}, corresponding to the record solution. The algorithm of the bee ρ ("scout") carries out the search in the whole admissible area. In the algorithm BCO, for each bee a decision is made, either to continue to complete the corresponding solution or to carry out a new search. In our algorithm for this purpose a certain part of the set of bees is used. The current set of solutions S is updated by replacement of the best and the worst solutions by the new ones. The parameters α and β also change subject to S and depend on subsets $S_1 \subset S$ and $S_2 \subset S \setminus S_1$, which are constructed by special rules. The set S_1 consists of α best solutions and S_2 consists of β worst solutions obtained in the previous iteration. For improvement of search in the algorithm of bee ϕ, the variable neighborhood search [3, 6] is used.

Let us describe the proposed algorithm.

Local search Bee Colony Optimization Algorithm (LBCO)

Preliminary step.

The initial value of quantity of bees, of a type of neighborhood and other parameters are defined. The formation of initial set of admissible solutions S is made.

Iteration $k, k \geq 1$. Until a stopping criterion is fulfilled do the following steps.

Step 1. Assessment of quality of solutions.
For each $z \in S$, calculate $F(z, X)$; choose a record value F_{rec} and a vector z_{rec} corresponding it.

Step 2. Definition of parameters of a new search.

Step 2.1. Generate a set S_1 of solutions, $S_1 \subset S$. Estimate a value of parameter α by it.

Step 2.2. Generate a set S_2 of solutions, $S_2 \subset S \setminus S_1$. Estimate a value of parameter β by it.

Step 3. Algorithm of a bee ϕ.

Step 3.1. Find α solutions from a neighborhood of current z_{rec}.

Step 3.2. Replace elements of set S_1 with solutions found on Step 3.1.

Step 4. Algorithm of a bee ρ.

Step 4.1. Find β solutions from all admissible area.

Step 4.2. Replace the elements of set S_2 with solutions found on Step 4.1.

Step 5. $k := k + 1$. Pass to the following iteration.

The set of solutions S_1, $S_1 \subset S$, is generated as follows:

$$S_1 = \{s \in S \mid \alpha > 0,5\},$$

where the parameter α is calculated by the formula:

$$\alpha(t) = \frac{L^s(t)}{\sum\limits_{j=1, j\neq s}^{B} L^s(t)}, \quad s \in S,$$

where
S is an initial set of vectors on the iteration t,
$L^s(t) = \max\Big\{(G^s(t) - \gamma\overline{G(t)}), 0\Big\}$ is a characteristic of solution s,
$\gamma \in (0; 1)$ is a coefficient,
$\overline{G(t)}$ is an average value of quality of all sources.

In the set of solutions S_2, $S_2 \subset S \setminus S_1$, those vectors are included which have the value of parameter $\beta < 0,5$:

$$S_2 = \{s \in S \setminus S_1 \mid \beta < 0,5\},$$

where the parameter β is calculated using the formula:

$$\beta = \exp\Big\{-\frac{\sum\limits_{i=1}^{B} L^s(t)}{2\sigma^2}\Big\}, \ \sigma \in (0; 1).$$

3 Computational Experiment

Computational experiments were carried out with an objective of approbation of introduced algorithms. The comparative analysis of their work with the algorithms of the ant colony (ACO) [4] on known test examples of various structure is executed.

It is necessary to note, that it was found that all problems from the library OR-Library [1] were easy both for ACO, and for LBCO. For all such problems the optimal solutions have been found, thus, for example, the CPU time for the first three classes of tests ($m \leq 50, n = 50$) was not more than 0,06 minutes.

On test examples from the electronic library of the Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Science [14], the algorithm LBCO developed by us has shown the best results in comparison with the algorithm of the ant colony from the point of view of the CPU time. The relative error of both algorithms was no more than 1,4 %, but the average time of finding of the optimum solution for LBCO was 8,13 minutes, and for ACO it was 13,44 minutes.

The experiment has shown, that the algorithm of a bee colony alongside with other heuristics can be successfully applied to location problems for finding approximate solutions. It is also interesting to use the proposed algorithm as a way of finding initial approximation for exact algorithms.

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