About optimal boundary control of string oscillations with nonlocal boundary conditions

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This paper is dedicated to the problem of boundary control of string vibrations at one end in the presence of nonlocal boundary conditions of Bitsadze-Samarsky type. We consider the oscillations of an elastic string with length l described by the one-dimensional wave equation

$$u_{tt}(x,t) - u_{xx}(x,t) = 0.$$

Nonlocal boundary conditions are prescribed any of the following four formulas

$$u(l,t) = -u(\overset{\circ}{x},t),\ u(l,t) = u(\overset{\circ}{x},t),$$

$$u_x(l,t) = -u_x(\overset{\circ}{x},t),\ u_x(l,t) = u_x(\overset{\circ}{x},t),$$

where $\overset{\circ}{x}$ is some internal point of the string. We consider the control by the displacement at the left endpoint $u(0,t) = \mu(t)$ or by force $u_x(0,t) = \mu(t)$ (ie, 8 different tasks). The task is to transfer the oscillating system from an arbitrarily given initial state

$$u(x,0) = \varphi(x),\ u_t(x,0) = \psi(x)$$

to the final state

$$u(x,T) = \widehat{\varphi}(x),\ u_t(x,T) = \widehat{\psi}(x)$$

in time T. So the first question arises about the controllability of the system, namely, under what restrictions on the initial and final functions we can find the control which solves our problem. In this paper we prove four theorems that formulate the necessary and sufficient conditions under which the boundary control can be found.

Then one can consider the optimization problem: among all the solutions of the boundary control we select the one, at which the minimum of the boundary energy integral $\int_0^T (\mu'(t))^2\,dt$ in case of displacement control and $\int_0^T (\mu(t))^2\,dt$ in case of force control is achieved. For each of eight problems optimal boundary control is found in explicit analytical form.

Incidentally occured problems of optimal boundary control of string vibrations at one end in the presence of a given mode at the other end are also of interest. These problems generalize previously discussed cases of the fixed end or free end.

Considerations are firstly given in terms of classical solutions from C^2, and then in the sense of generalized solutions of Sobolev space W_2^1.

References

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