

Ball motion with rough surfaces impacts.

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Some tasks about movement of a sphere by inertia are examined: between two Parallel Planes, inside Sphere and inside a Circular Cylinder. It's considered that conditions of rolling without slipping are satisfied (as a result of impact) — tangent speed of a Sphere's contacting point is equal to zero. It's proved, that for all this cases the motion in the limit comes to established speed regime: the angular velocity of a sphere tends to constant value, and the velocity of the center becomes periodic (for Sphere and Cylinder). In some cases position's and orientation's coordinates of a Sphere comes to the steady regime.

Also the control of the ball motion is considered (in the case of parallel planes), with which it is possible to give the exact coordinates of the ball.

Model of impact. Let us consider homogeneous ball with radius a and with a unit mass. The principal central moments of inertia are \mathbf{J} . The ball motion is by inertia and is fixed by smooth surface. Let the ball impacts the surface at the point \mathbf{P} . Let us introduce the following designations: γ — unit normal to the surface at the point \mathbf{P} directed inside the area valid for motion of the ball, ω — the angular ball velocity; V_c — the velocity of its center \mathbf{C} . We consider that the ball impact is take place on the model of completely rough surfaces and after the impact to the surface a tangent component of the ball velocity is zero. So $V_p = V_c - [\omega, a\gamma]$. An impact is absolutely elastic, i.e. the kinetic energy of the ball at impact is preserved: $(V_c^2 + J\omega^2)^+ + (V_c^2 + J\omega^2)^- = h$, where h — is double constant of energy integral. Assume that the moments of rolling and spinning friction at the impact are absent. So on the impact kinetic moment of the ball (respect to this point) is

preserved: $(a[\gamma, V_c] + J\omega)^- = a[\gamma, V_c] + J\omega)^+$.

The ball motion between two parallel planes. As a result of the examination is showed that ball motion is like the rolling without slipping on a plane. The angular velocity is directed along a constant vector. The projection of the ball center on a horizontal plane moves along a fixed line directed along the vector. If the planes will move with constant velocity in opposite directions, the position of the ball and its coordinates can be controlled by changing the absolute value of velocity, which is also obtained in this work.

References

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