Mathematical model of traffic flow with two time scales

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The paper proposes to consider a microscopic model of traffic flow on the example of a major city. The model is based scheme following the leader [?], the calculations are similar to the predictor-corrector scheme [?] in the theory of differential calculus. The model takes into account the various changes of traffic flow: speed characteristics of vehicles, narrowing roads, changing traffic lights, random start vehicles with a given destination, the transit flow of cars through the city, etc.

As a topographic base model city transport system we consider non-oriented graph, the edges of a road or highway junctions are a crossroads. Scheme of the transport network consists of edges as road and vertices as crossroads. The model works in cycles. Tact time of the system is denoted Δt, and assume comparable to the response time of the driver, i.e. 1–2 seconds. The vehicle is a point on the edge of the graph, which has two parameters: x_i is position on the edge of the graph and v_i is rate.

Since the motion is determined by the leader, then we consider it a known speed: it aims at maximum or zero before the intersection with red light. Then we can predict the speed of other traffic participants:

$$x_i(t) + v_i(t+\Delta t)\Delta t - x_{i+1}(t) - \\ - v_{i+1}(t+\Delta t)\Delta t = Su \quad (1)$$

where $x_i(t)$ and $v_i(t)$ - this position and vehicle speed at time t, u - the minimum allowable distance between the machines, S - parameter imposes a limitation on the maximum rate, M - number of vehicles on the road (edge), $i = 1, \ldots, M$.

Please note that the vehicle starts moving at time t, is wedged in a stream of cars, if it does not interfere with other vehicles, i.e. exit is open and the speed of travel is sufficient to avoid a collision was accomplished. Also, a vehicle that ends the movement on the highway, ie, having a final destination on the edge of the graph affects the motion of other vehicles involved in the movement. It is therefore desirable to consider smaller moments $\Delta\tau$, considering the solution of (**??**) "predictor" to the decision, which at time t tend vehicles. In this case, the time corresponding to one step of the system will $\Delta\tau$, whose value is chosen by several orders of magnitude smaller than Δt. Thus, it is proposed, based on predicted values $\Delta\tau$ obtained in (**??**), to calculate improved values $\Delta\tau$ as follows:

$$x_i(t+\Delta\tau) = x_i(t) + v_i(t)\Delta\tau + \\ + \alpha(x_i)\left(\frac{v_i(t+\Delta t) - v_i(t)}{\Delta t}\right)\Delta\tau^2 \quad (2)$$

$$v_i(t+\Delta\tau) = v_i(t) + \\ + \alpha(x_i)\left(\frac{v_i(t+\Delta t) - v_i(t)}{\Delta t}\right)\Delta\tau \quad (3)$$

where $v_i(t+\Delta t)$ results from (**??**), and $\alpha(x)$ - a ratio simulates narrow carriageway, or limiting speed cornering. Further, taking over t is the time $t + \Delta t$, the calculation will continue in the next step. Note that formula (**??**) describes only one possible strategy, in which the vehicle is focused only on the leader, the vehicle in front, or a traffic light. We call this strategy "selfish". We can suggest other strategies, when the vehicle is guided not only by the leader, for example, when the predictor is working according to the expression:

$$- x_{i-1} + v_{i-1}\Delta t + 2(x_i + v_i\Delta t) - \\ - x_{i+1} - v_{i+1}\Delta t = 0 \quad (4)$$

where $i = 1, \ldots, M$.

For example, (??) simulates the movement of military equipment in the convoy. Knowing the speed of the leader, we can consider (??) as a tridiagonal system of equations. This "friendly" strategy allows you to send information about a traffic jam upstream. Note that, apart from the right side of (??) for a negative number, for example, fold, we have a range of tasks that lie between the "selfish" and "friendly" models.

Obviously, the results of the calculations will be the velocity and position of all vehicles.

Figure 1. Moving vehicles from one road to another

To verify compliance with the proposed model real situations that arise when moving vehicles, naturally arises the need to build a typical test situation. Natural situation arising from the very first lines of language models, is the situation (Fig. 1), in which vehicles are on the road (edge) toward the top, cross it and continue the movement was already on the next road. The system consists of two edges (Fig. 1). At this stage the lights are not placed at the top, which allows vehicles to move from one road to another without stopping. Vehicles are generated randomly at the first rib. As a destination point is chosen on the second rib. This situation makes it possible to verify the correctness of the scheme in the calculation of the projected positions of vehicles at discrete intervals of time.

By adding traffic lights at the top of the graph, go to the modeling of the situation (Fig. 2), which became known as "the work of the T-crossroads". The difference between this case and the previous addition of additional edges, is the presence of traffic lights at the intersection (at top). In this situation, the vehicles "are generated" on the first and second road, and in order to reach the end of the third. Red and green traffic lights are burning the same amount of time.

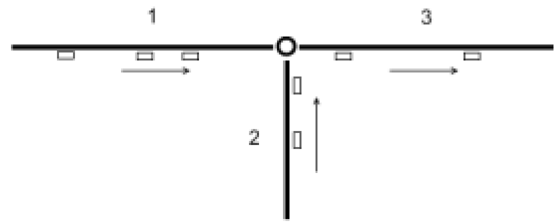

Figure 2. T-crossroads

Table 1.

Comparison of strategies

M	$T1_{avg}/T2_{avg}$
10	1.313
20	1.274
30	1.23
40	1.192
50	1.13
60	1.08

This situation makes it possible to simulate the formation of traffic jams and calculate the critical density of vehicles in which there are traffic jams. Note that in this situation shows the advantage of statistically "friendly" model to the "selfish". To obtain comparative results have been used 10000 tests on each of the two strategies (Table 1). In this busy hearse, ie number of vehicles generated by M from 10 to 60 on the edge. Thus obtained comparative results relative $T1_{avg}/T2_{avg}$, where $T1_{avg}$ - average execution time using the first strategy, and $T2_{avg}$ - respectively second.

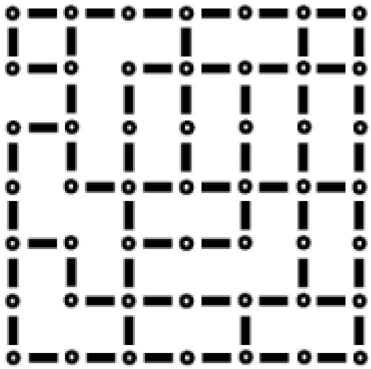

Figure 3. Region

The following situation is more complicated (Fig. 3). In this example, we consider a lattice of 49 intersections (vertices), some of which are connected by roads (edges) with others. Vehicles are generated at random on random roads. Each of them is given a random destination, and calculates the optimal route to reach it. Then turn the vehicle in motion. By varying the number of vehicles generated at each step of the system, you can load different network number of vehicles, identifying sites, primarily affected by the formation of traffic jams. We call such a lattice region.

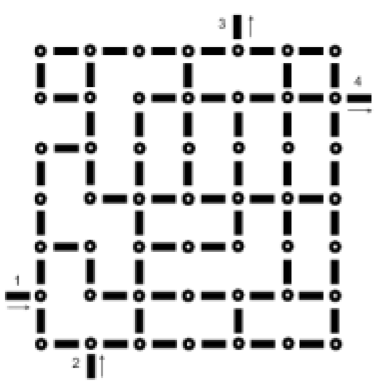

Figure 4. "The flow" through the region

This model allows us to pose the problem of finding the optimal operation of traffic lights to a maximum of "pumping" of vehicles across the region. Without loss of generality we define the time of traffic lights function of phase shift relative to the fundamental oscillation frequency ω, $\beta_1, ..., \beta_P$, where P is the number of traffic signals in the region. On certain routes or areas in the region should transit transport (Fig. 4), the average speed is at the intersection of the region is the objective function, we need to maximize value by choosing the phases.

Table 2.

	t_{vmin}	t_{vmax}	$v0_{avg}$	$v1_{avg}$
1	24.5	81.37	10.7	12.3
2	34.8	69.4	9.5	10.7
3	48.56	43.74	8.43	9.52
4	67.13	25.1	7.16	8.04
5	85.79	10.88	6.24	6.89
6	111.2	1.4	5.1	5.57
7	146.4	0	4.78	5.26

As a result of numerical experiments using a random search for finding such a set of $\beta_1, ..., \beta_P$, the average rate of passage through the region $v1_{avg}$ after finding the optimal set of $\beta_1, ..., \beta_P$ would be greater than the average rate of passage through the region $v0_{avg}$ to search, it was shown that significantly improve the average speed of traffic congestion with a small (Table 2), for example, the number of vehicles from 1 to 4 at 100 meters. The same results were obtained depending on the time spent in a standing position t_{vmin}, and at a maximum speed t_{vmax}, from busy roads. This task is in some sense, the development of the problem of finding the so-called "green wave".

The question of the class of the optimization problem is now open.

Solution discussed specific problems of modeling and optimization naturally leads us to demand maximum performance computing. The calculations show that the movement in real-time vehicles in an amount of 100000 units simulated a single processor, but the problem of simulation of large dimension, and especially optimization problems require increasing computation speed by orders of magnitude.

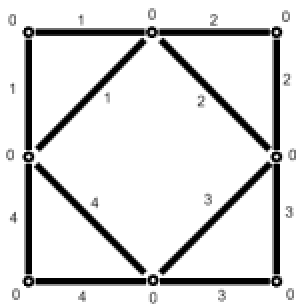

Figure 5. Scheme of parallelism

For example, if it comes to managing traffic, the number of calculations needs to be large, and they should be significantly faster.

The first and most obvious natural scheme of parallelism, threading it across regions where the calculations take place independently, and the exchange is limited to the transfer of control of vehicles crossing the border region. Figure 5 shows the scheme of parallelism where the index of an edge or a vertex corresponds to the processor number (index peaks, the number of the processor is responsible for traffic lights). It is obvious that the regions should be organized in such a way that the contacts between the regions were minimal, i.e. number of contact peaks was minimal.

As the experience of computing the number of controlled vehicles, the on-like simulations can reach numbers of the order and time of calculation of variations of motion is much smaller than the time of those options really are. Also, it seems promising to transfer a portion of mass calculations on GPUs, choosing the optimal size of the region or in a special way to parallelize calculations on conventional and graphics processor.

References

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[2] V. S. Ryabenkii and S. V. Tsynkov. *A Theoretical Introduction to Numerical Analysis.* CRC PRESS, 2006.