Pseudoregular plane covering by mobile sensors with adjustable sensing and communication ranges*

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Abstract

In the well studied regular covers of a plane region by disks, the region is tiled by the equal regular polygons (tiles), and all tiles are covered equally. In this case the centers of disks are placed in the certain points of each tile. In the sensor network the disk is usually a sensing area of the sensor. However sometimes it is impossible to place the sensors in the certain points, and they are distributed arbitrarily over the region. In the last case it is hard to organize the network and to analyse its efficiency. We proposed a new method to arrange the pseudoregular covers using different grids and show the way how to gain the sensor's mobility. We compare the lifetimes of mobile and static wireless sensor networks in case when the sensors are equally distributed over the region.

1 Introduction

Wireless sensor network (WSN) is presented by the set J, $|J| = m$, of mobile sensors with adjustable sensing and communication ranges, which are distributed over the plane region O of space S. Each sensor being in active mode consumes its limited energy for sensing, communication and movement. In a sleep mode sensor preserves its energy. Let the monitoring and communication areas of every sensor are the disks of certain radii with sensor in the center [1, 6, 7, 8]. The region O is covered if every its point belongs to at least one sensing disk. Arc (i, j) belongs to communication network, if sensor j is inside communication range of sensor i. WSN's lifetime is a number of time rounds during which region O is covered by the connected active sensors. The problem is to maximize a lifetime of WSN. This problem is very complex, and even special cases remain NP-hard [5]. Our goal is to get advantage of sensor's mobility in comparison with static sensor models in class of regular covers [7, 8]. There are few papers devoted to the sensor's mobility [2, 4], and our results are new and promising.

Suppose that each sensor has an energy storage $q > 0$. For any sensor, a sensing energy consumption per time round depends on a sensing range r (radius of sensing disk) and equals $SE = \mu_1 r^a$, $\mu_1 > 0$, $a \geq 2$; a communication energy consumption per time round depends on distance d and equals $CE = \mu_2 d^b$, $\mu_2 > 0$, $b \geq 2$; and energy consumption per time round during sensors motion depends on a speed v and equals $ME = \mu_3 v^c$, $\mu_3 > 0$, $c > 0$. We suppose, for the sake of simplicity, that during the motion a sensor do not consumes its energy for sensing and communication.

Let some regular grid, which tiles region O, is given. In a regular cover the sensors as usually placed in the grid nodes and cover some disk area (if grid node has number i, then we call the disk it covers as *disk* i). Let the sensors are distributed *uniformly* over the region O, and parameter $a_{ij} = 1$ if i is a closest grid node to the sensor j and $a_{ij} = 0$ otherwise. Denote the set $J_i = \{j \in J | a_{ij} = 1\}$, and we reasonably suppose that the sensor $j \in J_i$,

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if it is active, must cover the disk i. Then if sensor j is located on a distance r away from the grid node i, then it must increase its sensing range by r in order to cover the disk i. Moreover, if the distance between the node i and sensor $j_1 \in J_i$ is r_1, and the distance between the node k and sensor $j_2 \in J_k$ is r_2, then in order to guarantee a communication between the sensors j_1 and j_2 it is necessary to increase the communication ranges of j_1 and j_2 by at least r_1+r_2 units. Since the sensors are mobile, then every sensor $j \in J_i$ can move towards the node i during some time rounds in order to be nearer to i. For the sake of simplicity, we suppose that the speed of every sensor is 0 or v. Therefore, if sensor $j \in J_i$ moves, then the speed is v and the direction is towards the grid node i.

In [3] we've presented approach which permits to compare the static and mobile regular covers which use triangular grid. In this paper we continue our research for rectangular grid and compare the new results with the results we got in [3]. In the next section we briefly remind the results in [3].

2 Triangular regular grid

In [3] we considered two cases: when grid is fixed and when grid is transposed. We call the last one as a free grid.

In the first case region O is tiled by the regular triangles (tiles) of side $R\sqrt{3}$. These triangles form a regular grid with the set of grid nodes – vertices of triangles. If the sensors are distributed uniformly, then we shown in [3] that the lifetime of mobile WSN equals

$$\Lambda_T^\delta \approx \sum_{k=1}^{\lfloor \frac{R\sqrt{3}}{2v} \rfloor} \frac{(q - l_k \mu_3 v^c) N_k}{E^s(k,l_k) + E_T^c(k,l_k)}, \qquad (1)$$

where sensing energy consumption $E^s(k,l) = \mu_1(R + (k-l)v)^a$, communication energy consumption $E_T^c(k,l) = \mu_2(R\sqrt{3} + 2(k-l)v)^b$, $N_k \approx m\pi(2k-1)v^2/S$, $K = \lfloor \delta/v \rfloor$ ($\lfloor A \rfloor$ is the integer part of A), $\delta = R\sqrt{3}/2$, $l_1 = 0$ and

$$l_k = \arg\max_{0 \le l \le k} \frac{q - l\mu_3 v^c}{E^s(k,l) + E_T^c(k,l)}, \ k \ge 2.$$

In free grid we suppose that the number of sensors N_k in $J_i^k = \{j \in J_i | \delta_{k-1} < d_{ij} \le \delta_k\}$, where $\delta_k = k \cdot v$ and d_{ij} is the distance between the sensor j and grid node i, is *sufficiently great* for each $k = 1, \dots, K$. If the grid is wandering, then we proved in [3] that the lifetime is this case is

$$\Lambda_T^v \ge \left(\frac{3R^2 m\pi}{4S} - \frac{\mu_1(R+v)^a + \mu_2(r\sqrt{3}+2v)^b}{q} \right) \cdot$$

$$\frac{q - \mu_3 v^c}{\mu_1 R^a + \mu_2 (R\sqrt{3})^b} + \frac{6mR^2}{25S} \frac{q - 2\mu_3 v^c}{\mu_1 R^a + \mu_2 (R\sqrt{3})^b}$$

3 Rectangular regular grid

Let's consider again two cases: grid is fixed and grid is transposed. Let region O is tiled now by the equal squares of side $R\sqrt{2}$. These squares form a regular grid. If all sensors have the same sensing range R and they are equally placed in the grid nodes, then this cover we called $R1$. In cover $R1$ each quadruple of neighbor disks of radius R with centers in the nodes of square has one common point in the center of tile. In cover $R1$ each active sensor, located in the node i, must cover a disk of radius R centered in the node i (*disk i*). Then the sensing energy consumption of every sensor equals $SE = \mu_1 R^a$. The communication distance for each sensor in cover $R1$ is $R\sqrt{2}$, hence the communication energy consumption per time round is $CE = \mu_2(R\sqrt{2})^b$, and then the lifetime of any sensor is $t_{R1} = q/(\mu_1 R^a + \mu_2(R\sqrt{2})^b$. Since the number of grid nodes in $R1$ is $N_{R1} \approx S/(2R^2)$, then the lifetime of regular cover $R1$ equals

$$L_{R1} \approx \frac{t_{R1} m}{N_{R1}} \approx \frac{2qm}{S(\mu_1 R^{a-2} + \mu_2 R^{b-2} (\sqrt{2})^b)}.$$

If the sensors are distributed uniformly over O, then the sensors inside the *square i* with center in the node i and the sides at the distance $\delta = R/\sqrt{2}$ from the center, are in the set J_i. Let's consider the concentric circles of radii $\delta_k = k \cdot v$, $k = 1, \dots, K$, with the center in some grid node. Any sensor

$$j \in J_i^k = \{j \in J_i | \delta_{k-1} < d_{ij} \le \delta_k\}$$

could reach the grid node i by at most k time rounds.

Since resource of each sensor is limited by q, then if any sensor $j \in J_i^k$ moves l time rounds and, as a result, consumes its energy, then, taking into account the remainder sensor-node distance $(k-l)v$, it can be active during

$$t_k(l) = (q - l\mu_3 v^c)/(\mu_1(R + (k-l)v)^a +$$

$$\mu_2(R\sqrt{2} + 2(k-l)v)^b)$$

time rounds. Function $t_k(l)$ is concave, then one can find $l_k = \arg\max_{0 \le l \le k} t_k(l)$ by $O(\log_2 K)$ time complexity. Since the sensors are distributed uniformly, then there are $N_k \approx m\pi(2k-1)v^2/S$ sensors in every set J_i^k. Let the prime active sensors are initially located in J_i^1, and we suppose that they do not move and are active during

$$L_1 \approx qN_1/(\mu_1(R+v)^a + \mu_2(R\sqrt{2} + 2v)^b)$$

time rounds. During this time L_1 the sensors in J_i^2 could move towards the grid node i, and then they can be active during

$$L_2 = N_2 \max_{0 \le l \le \min\{2, L_1\}} t_2(l)$$

time periods. Therefore, during time $\Lambda_{k-1} = \sum_{l=1}^{k-1} L_l$ the sensors in J_i^k could move to the grid node i, and then they can be active during

$$L_k = N_k \max_{0 \le l \le \min\{k, \Lambda_{k-1}\}} t_k(l)$$

time rounds. Setting $l_1 = 0$, the lifetime of such mobile WSN equals

$$\Lambda_R^\delta \approx \sum_{k=1}^{\lfloor \frac{R}{v\sqrt{2}} \rfloor} \frac{(q - l_k\mu_3 v^c)N_k}{E^s(k, l_k) + E_R^c(k, l_k)}, \qquad (2)$$

where $E_R^c(k,l) = \mu_2(R\sqrt{2} + 2(k-l)v)^b$.

Let's compare (1) and (2). The k-th summand in (1) is always less than the k-th summand in (2). But the number of summands in (1) is greater than in (2). Then Λ_T^δ can be both more than Λ_R^δ and vice versa. For example, if $l_k = k$, $v = 1$, $R = 8$, $\mu_1 = 5$, $\mu_2 = 1$, $\mu_3 = 10$, $a = b = 2$, then

$$\Lambda_T^\delta \approx \frac{qN_1}{656,43} + \sum_{k=2}^{6} \frac{(q-10k)N_k}{512}$$

$$\approx \frac{m\pi}{448S}(713q - 14000)$$

and

$$\Lambda_R^\delta \approx \frac{qN_1}{582,25} + \sum_{k=2}^{5} \frac{(q-10k)N_k}{448}$$

$$\approx \frac{m\pi}{448S}(25q - 9400).$$

Therefore, if $q < 0,6686$, then $\Lambda_R^\delta < \Lambda_T^\delta$, else $\Lambda_R^\delta \ge \Lambda_T^\delta$.

In a free greed we suppose that the number of sensors N_k in J_i^k is sufficiently great for each $k = 1, \ldots, K$. If grid is wandering, then we may displace it several times without change the size (a new grid node i_n is relocated 2δ away from previous position i_{n-1} to the right and down), then WSN's lifetime can be increased as follows. Let us set $\delta = v$ and suppose that during the first time round, when a part of sensors in $J_{i_1}^1$ are active, other sensors in every set $J_{i_n}^1$, $n \ge 1$, move to the grid node i_n. The number of sensors in each set $J_{i_1}^1$, which are active during one (prime) time round, equals

$$n_1' \approx (\mu_1(R+v)^a + \mu_2(R\sqrt{2} + 2v)^b)/q,$$

and we suppose that $n_1' \le N_1$. These sensors don't move and the active ones must increase their sensing ranges by v. During the prime time round $N_1 - n_1'$ sensors in each set $J_{i_1}^1$ can reach the grid node i_1, and it is not necessary to increase their sensing ranges to cover the O. Moreover, each sensor in $J_{i_n}^1$, $n \ge 2$, can reach i_n during the first time round. The number of sensors in every set $J_{i_n}^1$, $n \ge 2$, is N_1, and the number of these sets (new grid nodes) is

$$n_2' \ge \lfloor R\sqrt{2}/(2v) \rfloor^2 - 1$$

($n_2' + 1$ is the number of disks of radius δ packed in a square of side $R\sqrt{2}$). Every sensor, located outside the sets $J_{i_n}^1$, $n \ge 1$, has two time rounds to reach the nearest grid node. The number of such sensors in a square is

$$n_3' \approx 2R^2m/S-(n_2'+1)N_1 \geq 2R^2m/S-R^2N_1/(2v^2).$$

Since $N_1 \approx m\pi v^2/S$, then the WSN's lifetime in this case is

$$\Lambda_R^v \approx 1+(N_1-n_1'+N_1n_2')\frac{q-\mu_3 v^c}{\mu_1 R^a+\mu_2(R\sqrt{2})^b}+ n_3'\frac{q-2\mu_3 v^c}{\mu_1 R^a+\mu_2(R\sqrt{2})^b}.$$

If one suppose inequality

$$(\mu_1(R+v)^a+\mu_2(R\sqrt{2}+2v)^b)/q \ll 1,$$

i.e. the resource of every sensor is big enough, then

$$\Lambda_R^v \approx \frac{m}{2S}\frac{2v^2\pi(q-\mu_3v^c)+4qR^2-\mu_3v^cR^2(8-\pi)}{\mu_1 R^a+\mu_2(R\sqrt{2})^b}.$$

For triangular grid under the similar assumptions, the lifetime is

$$\Lambda_T^v \approx \frac{m}{100S}\frac{75R^2\pi(q-\mu_3v^c)+24R^2(q-2\mu_3v^c)}{\mu_1 R^a+\mu_2(R\sqrt{3})^b}.$$

Set, for example, $v=1$, $R=8$, $\mu_1=5$, $\mu_2=1$, $\mu_3=10$, $a=b=2$ and let's compare Λ_T^v and Λ_R^v. In this case $m\Lambda_T^1/S = F_T^1 \approx (52q-267)/160$, $m\Lambda_R^1/S = F_R^1 \approx (131q-1587)/448$, and F_T^1 is always greater than F_R^1. If $R=2$, and the other parameters conserve their values, then $F_T^1 < F_R^1$ if $q > 13,48$, and $F_T^1 \geq F_R^1$ otherwise.

4 Conclusion

Mobility of the sensors is unquestionable advantage. But this additional option must be used optimally. We consider the triangular and rectangular regular covers of a plane area by equal disks and proposed two ways to explore sensor's mobility to construct the pseudoreqular covers. We evaluate these covers by estimating the low bound for WSN's lifetime and comparing with static networks. In [3] we've done a part of work for the triangular pseudoreqular covers. In this paper we continue our research for the rectangular pseudoreqular covers. We shown that any pseudoreqular cover could be preferable. It depends on the parameter's values.

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