ON A ZONAL FEEDBACK CONTROL PROBLEM IN DISTRIBUTED SYSTEMS

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1. Introduction

In the work, we investigate a problem of feedback control of lumped sources for objects with distributed parameters on basis of continuous observation over the phase state at certain points of the object. We propose an approach according to which the space of the values of phase states (phase space) at observed points is a priori partitioned into given subsets (zones). Feedback controls are chosen from a class of piecewise constant functions, and their current values are determined by a subset of the phase space which the current states of the object at its observed points belong to. Such feedback controls are called zonal in the work. We give a technique for obtaining optimal values of the zonal controls with the application of first order optimization methods. With this purpose, formulae for the gradient of the target functional in the space of zonal controls are obtained.

2. Problem statement

To illustrate the proposed approach to the investigation of the feedback control problem in distributed systems, consider the problem of control of plate heating by means of l lumped point sources. This process can be described by the following equation:

$$u_t = a^2 \Delta u + \sum_{j=1}^{l} \vartheta^j(t)\, \delta\left(x - \overline{x}^j\right), \qquad (1)$$

$$(x,t) \in \Omega \times (t_0, T],$$

where $\Omega \subset R^2$ is two-dimensional domain occupied by the plate. At the points $\overline{x}^j = \left(\overline{x}_1^j, \overline{x}_2^j\right)$ of the plate, there are heat sources with optimized power $\vartheta^j(t) \in V^j,\ j = 1,2,...,l$; V^j the set of admissible values of the j-th source; l given number of sources; Δ the two-dimensional Laplace operator; $\delta(.)$ the two-dimensional generalized delta function; a^2 the thermal diffusivity.

Let initial and boundary conditions be given, for example, in the following form:

$$u(x, t_0) = g_0(x) \in G_0,\ x \in \Omega, \qquad (2)$$

$$u(x,t)\big|_{x\in\Gamma_1} = g_1(x,t) \in G_1,\ t \in (t_0, T], \qquad (3)$$

$$\left.\frac{du(x,t)}{dn}\right|_{x\in\Gamma_2} = g_2(x,t) \in G_2,\ t \in (t_0, T], \qquad (4)$$

$$\Gamma_1 \cap \Gamma_2 = \emptyset, \quad \Gamma_1 \cup \Gamma_2 = \Gamma = \partial\Omega.$$

Here functions $g_i(.)\,,\ i = 0,1,2$ are given uncertainly, but their values belong to some given admissible sets $G_i,\ 0,1,2$ with known distribution functions $\Phi_{G_i}(g_i)\,,\ i = 0,1,2$ characterizing the distributions of possible values which the initial and boundary conditions may take on.

Assume that there are sensors installed at Npoints of the plate with coordinates

$$\tilde{x}^s = (\tilde{x}_1^s, \tilde{x}_2^s) \in \Omega,\ s = 1,...,N. \qquad (5)$$

These sensors realize operative observation and input information on the state of the heating process at these points into the control system determined by the vector:

$$\tilde{u}(t) = \left(\tilde{u}^1(t), ..., \tilde{u}^N(t)\right) = \tag{6}$$

$$= \left(u\left(\tilde{x}^1, t\right), ..., u\left(\tilde{x}^N, t\right)\right), \ t \in [t_0, T].$$

The considered problem of control of plate heating process consists in selecting current values of the powers of the sources according to the measured values of the states at the observed points of the plate

$$\vartheta^j(t) = \vartheta^j\left(\tilde{u}^1(t), ..., \tilde{u}^N(t)\right), \ \vartheta^j(t) \in V^j, \tag{7}$$

$$j = 1, ..., l, \ t \in [t_0, T],$$

$$\tilde{u}^s(t) = u(\tilde{x}^s, t), \ s = 1, ..., N, \tag{8}$$

so that to minimize a criterion of control quality given, for example, by the following functional:

$$J(v) = \int_{G_0} \int_{G_1} \int_{G_2} \int_{\Omega} [u(x, T; \vartheta, g_0, g_1, g_2) - u^*(x)]^2 \cdot$$
$$\cdot dx d\Phi_2(g_2) \, d\Phi_1(g_1) \, d\Phi_0(g_0) + \sum_{j=1}^{l} \int_0^T [\vartheta^j(t)]^2. \tag{9}$$

Here $u(x, T; \vartheta, g_0, g_1, g_2)$ is the solution to problem (1)-(4) corresponding to specifically chosen initial and boundary functions $g_0(x), \ g_1(x,t), \ g_2(x,t)$ and to admissible values of the control $\vartheta(t)$; $u^*(x)$ given function characterizing a desired distribution of the temperature at the final moment of time of the heating process.

Let the values of the phase states of the plate satisfy the following inequality

$$\underline{u} \le u(x, t; \vartheta, g_0, g_1, g_2) \le \overline{u}, \quad (x, t) \in \Omega, \tag{10}$$

under all possible admissible controls and initial and boundary functions

$$\vartheta(t) \in V, \ g_0(x) \in G_0, \tag{11}$$

$$g_1(x, t) \in G_1, \ g_2(x, t) \in G_2.$$

Partition the set of all possible values of the temperature $[\underline{u}, \overline{u}]$ into m intervals

$$[\underline{u}, \overline{u}] = \bigcup_{k=0}^{m-2} [u_{m-1}, u_m), \ u_0 = \underline{u}, \ u_m = \overline{u} \tag{12}$$

by the values $u_k, \ k = 0, ..., m$.

Piecewise constant values of the controls are chosen such that they depend on the states of the plate at the observed points, namely, according to which temperature interval the current temperature of the plate belongs to.

Let the controls be piecewise constant functions

$$\vartheta^j(t) = \vartheta^j_{i_1, i_2, ..., i_N} = const \tag{13}$$

when

$$u_{i_s - 1} \le u(\tilde{x}^s, t; \vartheta(t), g_0, g_1, g_2) < u_{i_s}, \tag{14}$$

$$i_s = 1, ..., m, \ s = 1, ..., N,$$

$$j = 1, 2, ..., l, \ \ t \in [0, T].$$

In the N-dimensional phase space of states $u(\tilde{x}^s, t), \ s = 1, ..., N$, sets (14) represent N-dimensional parallelepipeds, the general number of which is m^N.

It is clear that controls (13), as well as (7) assume the presence of feedback. In case of (13), the values of the powers of the controlled sources in the process of plate heating change only at the moments when the array of states at the observed points proceeds from one phase parallelepiped (14) into another.

The number of different values which the power of each source may take on is equal to the number of phase parallelepipeds defined by the inequalities (14), i.e. to m^N. The general number of optimized parameters

$$\vartheta = \left(\left(\vartheta^j_{i_1, i_2, ..., i_N}\right)\right), \tag{15}$$

$$i_s = 1, ..., m, \ \ s = 1, ..., N, \ j = 1, ..., l$$

is equal to lm^N. They define the behaviour of the sources under all possible states of the plate at the observed points which may occur under different admissible initial and boundary conditions and control actions.

Thus, the considered problem of control of plate heating process in a closed-loop fashion on a class of piecewise constant functions consists in optimizing lm^N-dimensional vector ϑ. The process of plate heating and, correspondingly, the value of target functional (9) depend directly on this vector.

3. Solution to the problem

To solve the zonal feedback control problem numerically, we use first order finite-dimensional optimization methods. With this purpose, we derive the formulae for the gradient of the functional in the space of the optimized parameters. Denote by

$$\Pi_{i_1,...,i_N}\left(\vartheta, g_0, g_1, g_2\right) \subset [t_0, T] \qquad (16)$$

a time interval when the phase state $\tilde{u}\left(t\right)$ is in $\left(i_1, ..., i_N\right)$ phase parallelepiped under chosen control $\vartheta\left(t\right)$ and functions participating at initial and boundary conditions $g_0\left(x\right),\ g_1\left(x,t\right),\ g_2\left(x,t\right)$. It is clear that

$$\bigcup_{i_1=1}^{m} ... \bigcup_{i_N=1}^{m} \Pi_{i_1,...,i_N}\left(\vartheta, g_0, g_1, g_2\right) = [t_0, T]. \qquad (17)$$

Consider the following conjugate boundary problem with respect to (1)-(4), (9):

$$\psi_t = -a^2\psi_{xx},\ \left(x,t\right) \in \Omega \times [t_0, T), \qquad (18)$$

$$\psi\left(x,T\right) = -2\left[u\left(x,T;\vartheta, g_0, g_1, g_2\right) - u^*\left(x\right)\right], \qquad (19)$$

$$\left.\psi\left(x,T\right)\right|_{x\in\Gamma_1} = 0,\quad \left.\frac{d\psi\left(x,T\right)}{dn}\right|_{x\in\Gamma_2} = 0. \qquad (20)$$

Here $\psi\left(x,t;\vartheta, g_0, g_1, g_2\right)$ is the conjugate variable – the solution to problem (18)-(20) under corresponding specifically chosen controls ϑ and initial and boundary functions $g_0\left(x\right),\ g_1\left(x,t\right),\ g_2\left(x,t\right)$, which the solution $u\left(x,t;\vartheta, g_0, g_1, g_2\right)$ to boundary problem (1)-(4) corresponds to. The value of this function at $t = T$ participates at initial condition (19) of the conjugate boundary problem.

The following theorem holds true.

Theorem. The components of the gradient of the functional in problem (1)-(4) in the space of piecewise constant controls (13), (14) for arbitrary control $\vartheta \in V$ under corresponding normalization are determined by the formulae:

$$\frac{\partial J\left(\vartheta\right)}{\partial\vartheta^j_{i_1,...,i_N}} =$$

$$= \int_{G_0}\int_{G_1}\int_{G_2}\int_{\Pi_{i_1,...,i_N}} \psi\left(\overline{x}^j, \tau; \vartheta, g_0, g_1, g_2\right)\cdot$$

$$\cdot d\tau d\Phi_{G_2}\left(g_2\right) d\Phi_{G_1}\left(g_1\right) d\Phi_{G_0}\left(g_0\right) + \qquad (21)$$

$$+2mes\left(\Pi_{i_1,...,i_N}\right)\vartheta_{i_1,...,i_N},$$

$$j = 1, ..., l,\ \ i_s = 1, ..., m,\ \ s = 1, ..., N.$$

The results of numerical experiments by the example of the solution to a feedback control problem are given.

4. Conclusion

The approach proposed in the work to building a control system with feedback for plate heating process can be used for controlling a lot of systems with distributed parameters described by other forms of mathematical models. An evident drawback of the proposed approach is highly computational work on determining the values of zonal controls. It is necessary to take into account that firstly, these computations need to be carried out preliminarily (at the stage of projecting the control system). Secondly, the development of up-to-date computer technology goes with high intensity. Thirdly, multiprocessor computers are widespread. It is easy to see that the formulae obtained in the work are easily parallelized.

All aforesaid allows to consider that the approaches similar to the one proposed in the work can find application in control systems built for technical objects and technological processes.

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