Projection algorithms for mathematical programming

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In work [1] iterative Fejer processes with projection operator are used to solve the system of linear equations (or inequalities) corresponding to necessary and sufficient conditions of an extremum for problems of linear programming. In the given work Fejer processes approach with small decreasing disturbances [2, 3] with projection operator for convex constrained mathematical programming problems with large quantity of linear constraints is considered. In these two frameworks it is also possible to develop parallel algorithms for decomposing large scale problems.

For an extreme problem $\min_{x \in V} h(x)$ where h is a convex function and V is considered as an intersection of linear constraints $V = \bigcap C_i$ the iterative process $x^{s+1} = \mathcal{F}(x^s + z^s), s = 0, 1, \ldots$ is considered where $\mathcal{F}$ is Fejer operator and $z^s \to 0$ is a decreasing small disturbance while $s \to +\infty$. In particular the vector z^s is taken as $-\lambda_s g^s$ where $\lambda_s \to 0$ are decreasing step multipliers, g^s is the gradient of function h at point x^s.

In program realisation of parallel algorithm it is suggested to use convex combination of projections onto separate sets of linear constraints C_i as Fejer operator $\mathcal{F}$. For the projection onto polyhedra the algorithm developed in [4] is used. Numerical experiments for consecutive version of the algorithm show almost linear dependence of major Fejer process number iterations from number of constraints of initial problem and give almost polinomial estimation of overall number of iterations in relation to number of constraints with an exponent close to 4.

The parallel algorithm is realised on Octave language (free software very similar to Matlab) and 'Message Passing Interface Toolbox for Octave' package (http://atc.ugr.es/javier-bin/mpitb) which is also available for Matlab. From the practical point of view execution time for the parallel algorithm depends on number of iterations in major iterative Fejer process and number of projection algorithm iterations. In particular execution time of the algorithm depends also on strategy of set V representation with overall number of constraints fixed. Figure 1 shows the parallel algorithm execution time for the different number of subsets C_i in set V representation for 2 small linear programming problems with 60 and 120 constraints respectively.

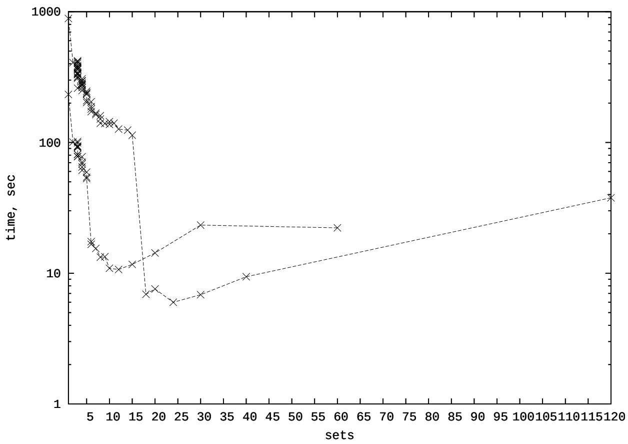

Figure 1: Parallel algorithm execution time for different set V representations.

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References

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