Numerical Calculus with Infinite and Infinitesimal Numbers and the Infinity Computer

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The lecture introduces a new methodology allowing one to execute numerical computations with finite, infinite, and infinitesimal numbers (see [1–12]). It is based on the principle 'The part is less than the whole' introduced by Ancient Greeks and applied to all numbers (finite, infinite, and infinitesimal) and to all sets and processes (finite and infinite). It is shown that it becomes possible to write down finite, infinite, and infinitesimal numbers by a finite number of symbols as particular cases of a unique framework different from those of the non-standard analysis and surreal numbers. The new approach gives possibility to introduce a new type of a computer – the Infinity Computer – able to operate numerically not only with finite numbers but also with infinite and infinitesimal ones (the European Patent Office has expressed its positive opinion with respect to the patent [2]).

The new methodology evolves Cantor's ideas in a more applied way and introduces new infinite numbers that possess both cardinal and ordinal properties as usual finite numbers. It gives the possibility to execute computations of a new type and simplifies fields of mathematics where the usage of the infinity and/or infinitesimals is necessary (e.g., divergent series, limits, derivatives, integrals, measure theory, probability theory, fractals, etc.). Numerous examples and applications are given. A number of results related to the First Hilbert Problem and to the Riemann zeta function are established.

In the following there are listed some operations that the Infinity Computer can execute and traditional computers are not able to perform and some of new areas of applications. The new approach allows:

– substitute symbols $+\infty$ and $-\infty$ by sets of positive and negative infinite numbers, to represent them in the memory of the Infinity Computer and to execute arithmetical operations with all of them numerically (not symbolically), as we are used to do with usual finite numbers on traditional computers;

– to substitute qualitative descriptions of the type 'a number tends to zero' by precise infinitesimal numbers, to represent them in the memory of the Infinity Computer, and to execute arithmetical operations with them numerically (not symbolically), as we are used to do with usual finite numbers using traditional computers;

– to calculate divergent limits, series, and improper integrals, providing as results explicitly written different infinite numbers, to be possibly used in further calculations on the Infinity Computer;

– to avoid appearance of indeterminate forms (e.g., in situations where it becomes necessary to calculate difference of two divergent series);

– to evaluate functions and their derivatives at infinitesimal, finite, and infinite points (infinite and infinitesimal values of functions and their derivatives can be also explicitly calculated);

– to study divergent processes at different infinite points;

– to introduce notions of lengths, areas, and volumes of traditional and blinking fractal objects obtained after infinite numbers of steps and compatible with traditional lengths, areas, and volumes of non-fractal objects and to calculate all of them in a unique framework.

The Infinity Calculator using the Infinity Computer technology is presented during the talk. Additional information can be downloaded from the page http://www.theinfinitycomputer.com

References

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