Usage of the Local Information in Lipschitz Global Optimization

Ya. D. Sergeyev*†, D. E. Kvasov*†, V. A. Grishagin†

*University of Calabria, Italy
†"N.I. Lobachevsky" University of Nizhni Novgorod, Russia

E-mails:
yaro@si.deis.unical.it
kvadim@si.deis.unical.it
vagris@unn.ac.ru

Global optimization problems, derived from high complexity industrial applications (see, e.g., [1, 2, 17, 19, 21, 22]), are often "black box" and determined by multiextremal objective functions. Solving efficiently this type of problems is a great challenge, since they present a high number of local minima, often with extremely different function values, and do not present a simple mathematical description of the global optima.

One of the natural and powerful (from both the theoretical and the applied points of view) assumptions on these problems is that the objective function has bounded slopes. In this case, the methods of the Lipschitz global optimization can be applied (see, e.g., [2, 5, 10, 17, 22, 23]).

This kind of global optimization problems is very frequent in practice. Let us refer only to the following examples: general (Lipschitz) nonlinear approximation; solution of nonlinear equations and inequalities; calibration of complex nonlinear system models; black-box systems optimization; optimization of complex hierarchical systems (related, for example, to facility location, mass-service systems); etc. (see, e.g., [2, 4, 5, 8, 15, 17, 22] and the references given therein).

As well known, the usage of the only global information about behavior of the objective function during its optimization can lead to a slow convergence of algorithms to global optimum points. Therefore, particular attention should be paid to the usage of a local information in global optimization methods, as well.

One of the traditional ways in this context (see, e.g., [5, 9, 10]) recommends stopping the global procedure and switching to a local optimization method in order to improve the solution and to accelerate the search during its final phase. Applying this technique can lead to some problems related to the combination of global and local phases, the main problem being that of determining when to stop the global procedure and start the local one. A premature arrest can provoke the loss of the global solution whereas a late one can slow down the search.

In this talk, another fruitful approaches will be discussed. The first one (the so-called local tuning approach, see [12,13,17,18,20,22]) allows global optimization algorithms to tune their behavior to the shape of the objective function at different subintervals of the admissible domain by estimating the local Lipschitz constants. The second one regards a continual local improvement of the current best solution incorporated in a global search procedure (see [16, 17]). These techniques become even more efficient when information about the objective function derivatives is available (see [3, 11, 14]). Several Lipschitz global optimization methods illustrating the above-mentioned concepts will be considered and compared numerically with some known algorithms (see [6, 7, 11, 16, 17]).

Acknowledgments

This research was supported by the following grants: Italian PRIN grant 2007-9PLLN7, Italian INdAM–GNCS grant 2009 for young researchers, Russian RFBR grant 07-01-00467-a and the grant 4694.2008.9 for the leading research groups awarded by the President of the Russian Federation.

References

[1] L. G. Casado, I. García, Ya. D. Sergeyev, Interval algorithms for finding the minimal root in a set of multiextremal non-differentiable one-dimensional

functions. *SIAM Journal on Scientific Computing*, vol. 24(2), pp. 359–376, 2002.

[2] Yu. G. Evtushenko, *Methods of Solving Extremal Problems and Their Application in Optimization Systems.* Nauka, Moscow, 1982. (In Russian).

[3] Yu. G. Evtushenko, M. A. Posypkin, Parallel methods for solving global optimization problems. In Proc. of the *4th International Conference "Parallel Computations and Control Problems"* (Moscow, 27–29 October 2008), pp. 18–39. (In Russian).

[4] Yu. G. Evtushenko, V. U. Malkova, A. A. Stanevichyus, Parallelization of the global extremum searching process. *Automation and Remote Control*, vol. 68(5), pp. 787–798, 2007.

[5] C. A. Floudas, P. M. Pardalos, Eds. *Encyclopedia of Optimization* (6 Volumes). Kluwer Academic Publishers, 2001.

[6] M. Gaviano, D. E. Kvasov, D. Lera, Ya. D. Sergeyev, Algorithm 829: Software for generation of classes of test functions with known local and global minima for global optimization. *ACM Transactions on Mathematical Software*, vol. 29(4), pp. 469–480, 2003.

[7] V. A. Grishagin, On convergence conditions for a class of global search algorithms. In Proc. of the *3rd All-Union Seminar on Numerical Methods of Nonlinear Programming* (Kharkov, 1979), pp. 82–84. (In Russian).

[8] V. A. Grishagin, Ya. D. Sergeyev, R. G. Strongin, Parallel characteristic algorithms for solving problems of global optimization. *Journal of Global Optimization*, vol. 10(2), pp. 185–206, 1997.

[9] R. Horst, H. Tuy, *Global Optimization – Deterministic Approaches.* Springer-Verlag, Berlin, 1996.

[10] R. Horst, P. M. Pardalos, Eds. *Handbook of Global Optimization*, vol. 1. Kluwer Academic Publishers, Dordrecht, 1995.

[11] D. E. Kvasov, Ya. D. Sergeyev, A univariate global search working with a set of Lipschitz constants for the first derivative. *Optimization Letters*, vol. 3(2), pp. 303–318, 2009.

[12] D. E. Kvasov, C. Pizzuti, Ya. D. Sergeyev, Local tuning and partition strategies for diagonal GO methods. *Numerische Mathematik*, vol. 94(1), pp. 93–106, 2003.

[13] Ya. D. Sergeyev, An information global optimization algorithm with local tuning. *SIAM Journal on Optimization*, vol. 5(4), pp. 858–870, 1995.

[14] Ya. D. Sergeyev, Global one-dimensional optimization using smooth auxiliary functions. *Mathematical Programming*, vol. 81(1), pp. 127–146, 1998.

[15] Ya. D. Sergeyev, V. A. Grishagin, Parallel asynchronous global search and the nested optimization scheme. *Journal of Computational Analysis and Applications*, vol. 3(2), pp. 123–145, 2001.

[16] Ya. D. Sergeyev, D. E. Kvasov, Global search based on efficient diagonal partitions and a set of Lipschitz constants. *SIAM Journal on Optimization*, vol. 16(3), pp. 910–937, 2006.

[17] Ya. D. Sergeyev, D. E. Kvasov, *Diagonal Global Optimization Methods.* Fizmatlit, Moscow, 2008. (In Russian).

[18] Ya. D. Sergeyev, D. E. Kvasov, F. M. H. Khalaf, A one-dimensional local tuning algorithm for solving GO problems with partially defined constraints. *Optimization Letters*, vol. 1(1), pp. 85–99, 2007.

[19] Ya. D. Sergeyev, P. Daponte, D. Grimaldi, A. Molinaro, Two methods for solving optimization problems arising in electronic measurements and electrical engineering. *SIAM Journal on Optimization*, vol. 10(1), pp. 1–21, 1999.

[20] Ya. D. Sergeyev, P. Pugliese, D. Famularo, Index information algorithm with local tuning for solving multidimensional global optimization problems with multiextremal constraints. *Mathematical Programming*, vol. 96(3), pp. 489–512, 2003.

[21] A. S. Strekalovsky, *Elements of Nonconvex Optimization.* Nauka, Novosibirsk, 2003. (In Russian).

[22] R. G. Strongin, Ya. D. Sergeyev, *Global optimization with non-convex constraints: Sequential and parallel algorithms.* Kluwer Academic Publishers, Dordrecht, 2000.

[23] A. A. Zhigljavsky, A. Žilinskas, *Stochastic Global Optimization.* Springer, New York, 2008.