A Markov Chain Monte Carlo method exploiting barrier functions with applications to optimization

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Randomized methods for control and optimization become highly popular [1, 2]. They often exploit modern versions of Monte Carlo technique, based on Markov Chain Monte Carlo (MCMC) approach [3, 4]. The examples of such MCMC methods as Hit-and-Run and Shake-and-Bake applied to various control and optimization problems are provided in [5, 6]. However the results are unsatisfactory for "bad" sets, such as level sets of ill-posed functions. The idea of the present paper is to exploit the technique, developed for interior-point methods of convex optimization [7], and to combine it with MCMC algorithms.

Suppose a convex set Q and corresponding barrier $F(x)$ are given. The first problem is to generate points x_i approximately uniformly distributed in Q. The method uses Hit-and-Run and local geometry of Q at the point x_i provided by Hessian of $F(x_i)$. The second problem is to minimize a linear function (c, x) over Q. We construct Dikin's ellipsoid centered at x_i and choose a random direction uniformly distributed in this ellipsoid. The stepsize in this direction is made according to boundary oracle [5, 6]. Such approach is well tailored for sets Q defined by linear matrix inequalities (LMI), which are widely used in control and optimization [8].

The results of numerical simulation are promising.

References

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