Application of Optimization Techniques for Reliability-oriented Design[1]

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The studies have shown that the engineering system parameters are subject to random variations and the variations may be considered as non-stationary stochastic processes. The conventional methods for choosing parameters (for parametric synthesis) generally do not take account of parameter production and field deviations of parameters from their design values. As a result, the engineering systems designed in such a manner are not optimal in the sense of their gradual failure reliability.
This paper proposes the approach and some algorithms for seeking a numerical solution of the parametric optimization problem with a stochastic (reliability) criterion. For solving this problem we use a so-called operational/parametric approach based on the computer-aided simulation of system capability and availability, parameters deviations and techniques of optimal parametric synthesis with respect to reliability criteria [1].
In general the optimal parametric design (synthesis) problem can be stated as follows.

Suppose that we have a system which depends on a set of n parameters $\boldsymbol{x}=(x_1,\ldots, x_n)$. We will say that system is acceptable if $\boldsymbol{Y(x)}$ satisfy the conditions (1):

$$\boldsymbol{a} \le \boldsymbol{Y} \le \boldsymbol{b}, \qquad (1)$$

where $\boldsymbol{Y}$, $\boldsymbol{a}$ and $\boldsymbol{b}$ are m-vectors of system responses (output parameters) and their specifications, e.g. $Y_1(\boldsymbol{x})$ - average power, $Y_2(\boldsymbol{x})$ -delay, $Y_3(\boldsymbol{x})$ - gain.

The inequalities (1) define a region D_x in the space of design parameters

$$D_x = \{\mathbf{x} \mid \boldsymbol{a} \le \boldsymbol{Y} \le \boldsymbol{b}\} \qquad (2)$$

D_x is called the tolerance margin domain (region of acceptability) for S. It is region in the input parameters space.

Let given the characteristics of random processes $\mathbf{X}(t)$ of system parameters variations, a region of admissible deviation - D_x and a service time T, find such a deterministic vector of parameter ratings (nominals) $\boldsymbol{x}_r=(x_{1r},\ldots,x_{nr})$ that the probability

$$P_r(\mathbf{x}_r, T) = P_r\{[X_1(t)-x_{1r},\ldots,X_n(t)-x_{nr}] \in D_x, \forall t \in [0,T]\}$$

be maximized.

Any optimization technique requires, first, a method of objective function calculation and, second, an extremum searching method which allows to find a solution with a minimum cost.

The practical algorithm of the stochastic criterion calculation is based on the conventional Monte Carlo method.

At the beginning, the random vector of parameters is generated (this vector means random manufacturing device realization), and then the internal parameters degradation is simulated using degradation model.
The Monte Carlo method approximates $P_r(x_r,T)$ by the ratio of number of acceptable realizations (falling in region D_x) - N_a to the total number of trials - N.

Unfortunately, often the region D_x is unknown. It is given only implicitly through system's equations and the systems response functions. If we do not know the region D, then a Monte Carlo evaluation of probability $P_r(\boldsymbol{x}_r,T)$ at particular nominal value $\boldsymbol{x}_r$ requires N system analyses for each trial set of parameter $\boldsymbol{x}_r$. Typically, hundreds of trials are required to obtain a reasonable estimate for $P_r(\boldsymbol{x}_r,T)$.

Optimization requires the evaluation of our probability $P_r(\boldsymbol{x}_r,T)$ for many different values of the nominal values of the parameters $\boldsymbol{x}_r$. Therefore to make practical the use of Monte Carlo techniques in statistical system design, it is necessary to reduce the number of system analysis required during optimization.

Particularly effective way to decrease total design time on the phase of modrelling and statistical optimization is to use modern supercomputing technologies and parallel processing techniques [2].

Note that evaluation of $\mathrm{extr}P_r(\boldsymbol{x}_r,T)$ requires a global optimization. The simplest method of global optimization is scanning (full enumeration) method. However, such method is considered computationally inefficient. The effective way to decrease optimization time is to use modern supercomputing technologies and parallel algorithms.

The nominal values of the schematic components $\boldsymbol{x}_n$ commonly used for engineering systems should lie in the predefined set of values as it is required by various standards and technical recommendations, it is sometimes more preferable to search the optimal vector inside the discrete set of values that conforms to the standards and lies in the acceptable region D_x.

Let us have the known internal parameters vector $\boldsymbol{x}_r \in D_x$. Therefore at the each point of discrete set $D_r^{in} = \{\mathbf{x}_r^{in} / \boldsymbol{x}_r \in D_x\}$ we need to find the $\hat{P}_r(\mathbf{x}_r^{in})$ estimate. The optimum nominal vector $\boldsymbol{x}_r$ we are looking for can be found as a solution of the following task

[1] This work was funded by the Grant 09-I-П2-03 (Basic Research Program of Presidium RAS № 2).

$$\boldsymbol{x}_r = \max_{\mathbf{x}_r} \hat{P}_r(\mathbf{x}_r^{in}) \qquad (3)$$

In the simplest case the solution can be found by complete check of each element of the set D_r^{in} with the probability estimation for each of them. The set D_r^{in} building can be implemented as a preliminary procedure that puts the element values to the database.

The optimum search process can be performed in parallel mode.

This algorithm can be presented as a two-level distributed process that requires *RN* processors for implementation (here *R* means the number of elements in the set D_r^{in}).

Note an analogous method would apply to the general optimization problem by using the regionalization (discretization) approach. Regionalization consists of dividing the tolerance box into a finite number of non overlapping regions D^J, to form a grid. Then, the center or midpoint c_j of each region D^j, is chosen to "represent" entire region.

The information on a variation of values of internal parameters can be presented as limits of their values

$$x_{i\,min} \le x_i \le x_{i\,max}, \quad i = \overline{1,n} \qquad (4)$$

The area in space of internal parameters assigned by relations (4), represents n-dimensional the orthogonal parallelepiped, which we shall name as a beam of tolerances B_∂:

$$B_\partial = \{\boldsymbol{x} |\ x_{i\,min} \le x_i \le x_{i\,max},\ i = \overline{1,n}\}$$

It is possible to define the area of acceptable values of parameters D_x by methods based on multivariate exploration of tolerance region B_∂. At multivariate exploration a beam (region) of tolerances B_∂ can be represented by a finite number of sampling points.

The choice of sampling points can be realized by splitting a tolerance box into a finite number of not intersected areas.

Let's divide a range of possible changes of each of internal parameters into some number of intervals $l^{(i)}$, $i = \overline{1,n}$, which we shall term as quantums. As a representative of quantum we shall select an option value x_i, laying in middle of quantum.

If the implementation of the system appears efficient or disabled at an option value x_i, we shall consider, that it is efficient or is disabled at all values of this parameter laying in given quantum.

Let's make a matrix of incompatible situations (combinations of representatives of quanta), understanding under a situation such status of the technical system, when each of its internal parameters possess the value, appropriate to a representative of particular quantum:

$$A = [\alpha_1, \alpha_2, \ldots, \alpha_v, \ldots, \alpha_R]^T,$$

where $\alpha_v = \{\mathbf{x}_i^v\}_{i=1}^n$ - vector of a *v*-th situation composed from representatives of quanta; $R = \prod_{i=1}^{n} l^{(i)}$ - total number of situations; the index T means transposition.

It is obvious, that in situations the discrete change of all parameters simultaneously is taken into account, and set of incompatible situations is sampled representation of a beam B_∂. Each of situations is some sampling point representing appropriate subregion of a tolerance box (quantum-neighborhood).

For each of *R* possible situations output parameters ***Y(x)*** is computed, condition (1) is tested and discrete set of parameter nominals $D_r^{in} = \{\mathbf{x}_r^{in} / \boldsymbol{x}_r \in D_x\}$ is formed.

The optimum nominal vector $\boldsymbol{x}_r$ we are looking for can be found as a solution of the task (3).

References

[1] O.V.Abramov, *Reliability-directed parametric synthesis of stochastic systems*. Nauka, 1992.

[2] O.V.Abramov and Y.V.Katueva, *Application of parallel computing techniques for stochastic optimization problems*, Proc. of the 5-th Asian Control Conference, Melbourne, 2004, pp.434-440.