Construction method of convexification approximations for the smooth functions with using of Shepard's operator

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Shepard's operator was proposed in [1] and it was adapted practically only for construction of algorithms for processing the cartographic images; infrequent papers on this subject are mentioned in a survey [2]. Shepard's function presents the relation of two rational-linear functions with the guaranted absence of poles and it easily can be construct for any dimension of being investigated dependence on "irregular grid"; by construction, this is a smooth infinitely differentiated function; in the simplest case is an interpolant. To calculate the approximating function at any point, we need to produce the polynomial quantity of arithmetic operations. The structure of Shepard's operator makes it possible easily to consider additional information about object, for example, the known values of derivatives in the nodal points.

In this report, we consider the construction method of convexification approximations of nonconvex functions with the possible reaching of convexification effect. We put the additional parameters in Shepard's operator. Control by the last ones makes it possible to reach the required properties of approximating functions. During the first step, the base collection of points in a variable set is generated. The values of functions and their derivatives are calculated in the points of a base collection for smooth functions. Further convexification approximation for derivatives is constructed with fixed values of control parameters. Next using obtained approximation in derivatives, the approximation of the original function is constructed. "Convexification effect" can be reach via the choice of approximate values of control parameters.

Examined methods make it possible to find extremum of nonconvex functions for various classes with apply of the standard methods of local optimization. In this case, optimal points of smoothed function are used as initial approximations for completing local descent in connection to original function. Produced computational experiments confirm fitness of the work for proposed methods on the test problems.

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References

[1] D. Shepard, *A two-dimensional interpolation function for irregularly-spaced data* // Proc. of the 23 ACM National Conference, ACM Press, New York, 1968, pp. 517-524.

[2] R. Caira, F. Dell'Accio, *Shepard-Bernoulli operators* // Mathematics of computation, 2007, V. 76, N. 257, pp. 299-321.