A Decomposition graph method for fixed charge minimal network flow problems

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We introduce a special class for a “Fixed charge minimal network flow problems (**FMNP**)h (sometimes, called “Network Synthesis Problems” in Wynants [3]) . **FMNP** have so many applications in telecommunication and transportation literature.

A formulation of **FMNP** is as follows.

$$\min \sum_{(i,j)\in A} \sum_{k\in K} d_{ij}^k f_{ij}^k + \sum_{(i,j)\in A} F_{ij} y_{ij} \tag{1}$$

subject to

$$\sum_{j\in N-\{i\}} f_{ij}^k - \sum_{j\in N-\{i\}} f_{ji}^k = \begin{cases} d_k & i = O(k) \\ 0 & \forall i \in N - \{O(k), D(k)\} \quad \forall k \in K \\ -d_k & i = D(k) \end{cases} \tag{2}$$

$$\sum_{k\in K} f_{ij}^k \le q y_{ij} \quad \forall (i,j) \in A \tag{3}$$

$$f_{ij}^k \ge 0 \quad \forall (i,j) \in A, \forall k \in K \tag{4}$$

$$y_{ij} \in \{0,1\} \quad \forall (i,j) \in A \tag{5}$$

FMNF is proved to be **NP**-hard in Garey [1]. Bacically, for complete graphs. And Mohri [2] showed that they can be solved for a kind of “series-parallel graph” in a strong polynomial time .

In this presentation, we show that they can be solved for more generalized graphs in a strong polynomial time. If we found a partition on a graph $G(N, A)$ i.e $(G_i(N_i, A_i))_i$ where G_i is a sub-graph which has a property like “series-parallel graph” and generate a graph $G^*(N^*, A^*)$ which is a forest by regarding each N_i as one node of N^*, they would be solved in a strong polynomial time.

Also, if we have time, we would like to talk about a hub (facility location) problem combined with **FMNP** for some tractable cases.

References

[1] M. R. Garey and D. S. Johnson, *Computers and Intractability:A Guide to the Theory of NP-Completeness.* Freeman, 1979.

[2] H. Mohri, *Network Design Problem for some graphs.* the Proceedings of International Conference of Numerical Analysis and Applied Mathematics 2004, p.269–272, Wiley-VCH, 2004.

[3] C. R. Wynants *Network Synthesis Problems.* (Combinatorial Optimization), Springer, 2000.