Continuous school timetable with duration of 4 and 5

A. M. Magomedov*, T. A. Magomedov†

*Daghestan state university, magomedtagir1@yandex.ru
†Daghestan state university, magomedtagir@mail.ru

1. Introduction. Let $L = \{1, \ldots, l\}$ be a set of teachers, and let $T = \{1, \ldots, t\}$ be a set of academic hours, acceptable for a school timetable, $N = \{1, \ldots, n\}$ — a set of classes ($n < l$); for each $i \in L$ such multiset ω_i, $|\omega_i| \leq t$, with elements from N is given that for each $j \in N$

$$\sum_{i=1}^{l} k_{i,j} = t,$$

where $k_{i,j}$ — the number of occurrences j in ω_i.

It is supposed that $(l \times m)$-matrix r (*timetable*) with elements from the set $\{0, \ldots, n\}$ was build and it satisfy to the following requirements: (1) multiset of nonzero elements in the ith row is equal to ω_i, $\forall i \in L$; (2) each column contain a set N and $l - n$ elements that are equal to zero.

It is considered the problem of the $(l \times m)$-matrix R (*continuous timetable*) existence that satisfy to the following requirements: (1) multiset of the elements in each *line* (row/column) of the matrix R is the same as in the corresponding line of the matrix r; (2) nonzero elements in each row of the matrix R are allocated *continuously* in the sense that all cells that are located between any pair of nonzero elements also contain nonzero elements.

Further by τ_p a family $\{\omega_i : |\omega_i| = p\}$ is denoted.

2. Timetable with duration of 4. In current section we use the same terminology as in [1].

Let $t = 4$. Create flow network S with nodes S_1 (*source*), $X = \{x_j : j \in N\}$, $Y = \{y_i : i \in L\}$ and S_2 (*sink*). We use notations $lo(e)$ $hi(e)$ for upper and lower bounds of the flow for the edge e correspondingly.

From S_1 to each node $x_j \in X$ create oriented edge e_j, $lo(e_j) = hi(e_j) = 2$;

from $x_j \in X$ to $y_i \in Y$ create $k_{i,j}$ parallel oriented edges $e_{i,j}$ and for each of them suppose $lo(e_{i,j}) = 0$, $hi(e_{i,j}) = 1$;

at last, from each node $y_i \in Y$ to node S_2 create oriented edge e_i, and suppose $lo(e_i) = 0$, $hi(e_i) = 1$, when a $|\omega_i| = 1$; $lo(e_i) = 1$, $hi(e_i) = 2$, when a $|\omega_i| = 2$; $lo(e_i) = hi(e_i) = 2$, when a $|\omega_i| \in \{3, 4\}$.

Theorem 1 *When $t = 4$ continuous timetable exists if and only if feasible flow exists in the flow network S.*

3. Timetable with duration of 5.

Theorem 2 *When $t = 5$ and a family τ_1 is empty, than for continuous timetable existence necessary and sufficient fulfillment of the following conditions:* (1) *family τ_4 is empty;* (2) *for a family $\tau_3 \cup \tau_5$ exists a set of distinct representatives.*

References

[1] L. Ford and D. Fulkerson. *Flows in Networks.* Princeton University Press: New Jersey, 1962.