A column generation method for the (r, p)-centroid problem

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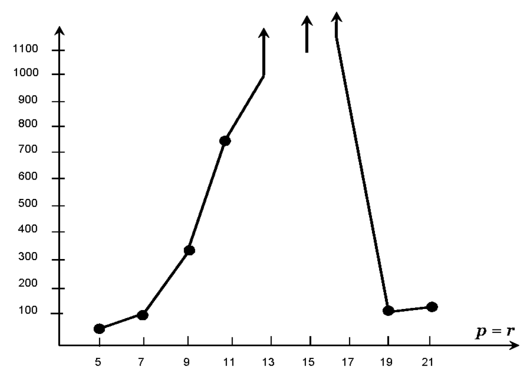
Much effort in the discrete location theory has aimed at developing insights concerning the classical models with one decision maker only. In this paper, we study a competitive model with two noncooperative decision makers: the leader and the follower. They compete to attract clients from the given market and wish to maximize their own profits. In fact, this is the noncooperative Stackelberg game. Following Hakimi [2], we call it *the (r, p)-centroid problem*. It is known that the problem is $\sum_2^p$-complete [3]. So, it is a more difficult problem than any decision problem in the class NP.

In order to find optimal or near optimal solutions we formulate this game as a mixed integer programming problem with exponential number of constraints and variables. Similar reformulation is used in [4] for partial enumeration approach but with a lot of additional constraints and variables. If we remove some constraints and variables we get an upper bound for the leader profit. In the column generation method we use "small" family of the constraints and variables and iteratively modify the family to decrease the upper bound. A maximum p-median problem is solved at each iteration of the method by the branch and bound algorithm. It is time consuming procedure. To reduce the running time at each iteration we use metaheuristics [1] to guess a global optimum and replace the p-median problem by a feasibility problem.

The developed method is coded in GAMS (General Algebraic Modeling System) and tested on instances from the benchmark library "Discrete Location Problems" (http://math.nsc.ru/AP/benchmarks/english.html). For all instances, we generate 100 points for possible location of facilities and clients. The elements of distance matrix are Euclidean distances between points in the two dimensional plane. The points are chosen at random with the uniform distribution in the square 7000×7000. All experiments are carried out at the PC Pentium Intel Core 2, 1.87 GHz, RAM 2 Gb, running under the Windows XP Professional operating system.

Our computational results show that the problem is tractable for $r, p \leq 5$ and 100 points. In figure we illustrate the dependence of running time (number of iterations) as a function of the parameters p, r for 50 points ($r = p$). We observe that the parameters have great influence on the running time of the method. We can find optimal solutions for small and large values only. So, we need some further improvements, for example, concerning for the feasibility problem. One of the future research directions is developing a metaheuristics for this optimization problem.



References

[1] E. Alekseeva, N. Kochetova, Y. Kochetov, A. Plyasunov *A Hybrid Memetic Algorithm for the Competitive p-Median Problem.* Proceedings of INCOM-09, Moscow, 2009.

[2] S. L. Hakimi. *Locations with spatial interactions: competitive locations and games.* In: P. B. Mirchandani, R. L. Francis (Eds.) Discrete Location Theory. Wiley & Sons. 1990. 439–478.

[3] H. Noltemeier, J. Spoerhase, H.-C. Wirth *Multiple voting location and single voting location on trees.* European J. Oper. Res. 2007 v. 181, 654–667.

[4] C. M. C. Rodriguez, J. A. M. Perez *Multiple voting location problems.* European J. Oper. Res. 2008. v. 191(2), 437–453.