Modelling of Lithuanian Private Consumption by Means of Dynamic Programming *

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Household consumption is the largest component of aggregate expenditure in most economies. In Lithuania it accounts for about 65% of spending. Therefore it is important for macroeconomists to be able to explain the determinants of consumer spending via a well-specified consumption function. Although literature on modelling consumption is large [2], in Lithuanian academic literature the analysis of household consumption, at macroeconomic level, is relatively scarce. Typically consumption is an integral part of a larger structural macroeconomic model [4],[5] and there is one recent publication devoted only for the modelling of consumption [3]. In the latter the authors use consumption as the error-correction type of model. Their results are quite close to the general ideas of Friedman's permanent income hypothesis. Modern theories of consumption are based on analysis of optimal consumption behaviour over time under constraint [1]. In equilibrium, a rational consumer chooses optimum levels of consumption in each period so as to maximize utility.

In this paper, we model household consumption from the perspective of the modern representative agent-based approaches. Household chooses a stochastic consumption plan to maximize the expected value of their time-additive nonlinear utility function subject to asset budget constraint

$$\max_{c_{t+i}} E_t \sum_{i=0}^{\infty} (1+\delta)^{-i} u(c_{t+i}) \tag{1}$$

subject to

$$R(A_{t+i} + y_{t+i} - c_{t+i}) = A_{t+i+1} \tag{2}$$

where $R = 1 + r$. E_t denotes the mathematical expectations operator conditional on information available at time t, δ is the rate of subjective time preference and acts like an interest rate. r is constant rate of real interest; c_t is consumption; A_t are assets apart from human capital; T is the length of economic life; $u(\cdot)$ is the one period-period utility function that is assumed strictly concave and time separable, and y_t are earnings which are stochastic. In addition, a non-negativity constraint on consumption must be imposed. We assume that this constraint is always fulfilled.

To estimate the structural parameter in equation (1) one needs to specify the functional form of the utility function. As the most popular utility function among the economists is CRRA type utility functions we use it the analysis. CRRA preferences have following form

$$u(c_t) = \begin{cases} \frac{c_t^{1-\sigma}}{1-\sigma}, & \text{if } \sigma \neq 1 \\ \ln c_t, & \text{if } \sigma = 1 \end{cases} \tag{3}$$

where σ is the coefficient of relative risk aversion.

There is no consencus about the size of discount factor $\beta = \frac{1}{1+\delta}$ and relative risk aversion coefficient. Common assumption is that $0 < \beta < 1, \sigma \geq 0$. Very often authors assume that latter coefficients are constant over time. In [1] authors fix the coefficient of relative risk aversion at the value of 1.5 and experiment with assumptions on the discount factor (0.99, 0.98, 0.96, 0.91).

Empirical analysis is conducted using annual Lithuanian data covering period from year 1995 to 2007. We estimate $\beta = 0.98$ corresponding $\sigma = 1.08$ and $\delta = 0.02$ for CRRA type utility function and will use them for further analysis.

The intertemporal separability of the objective function and the accumulation constraints allow us to use dynamic programming methods to solve the above problem, which can be decomposed into sequence of two-period optimization problems. As in [6] and [7] for dynamic programming we use the Bellman equation, which general form is

*The authors are grateful to the Lithuanian State Science and Studies Foundation for financial support (Project T-77/09).

$$V_t(A_t) = \max_{c_t} \left(u\left(c_t\right) + \beta E_t \left[V_{t+1}\left(A_{t+1}\right)\right)\right] \quad (4)$$

where $V_t(\cdot)$ a value function. The value function is stochastic as future income are uncertain and enters (4) as an expected value. The first-order conditions, so-called Euler equation, are obtained taking derivate of (4) with respect to consumption c_t subject to constraint (2).

This equation gives us a way of solving the original optimization problem. The idea is to start at the end and proceed to earlier times recursively. Suppose that the consumer's horizon ends at time T. The final period where a choice is made is in $T-1$. In the period T, the individual consumes the remaining wealth and the labour income, i.e., $c_T = A_T + y_T$, if we assume that there is no bequest, i.e., $A_{T+1} = 0$. The optimization problem at time $T-1$ is

$$\begin{aligned} V_{T-1}(A_{T-1}) = \max\nolimits_{c_{T-1}} \left(u\left(c_{T-1}\right) + \right. \\ \left. + \beta E_{T-1}\left[V_T\left(R\left(A_{T-1} - c_{T-1}\right) + y_{T-1}\right)\right]\right) \end{aligned} \quad (5)$$

subject to

$$A_T = R(A_{T-1} + y_{T-1} - c_{T-1}). \quad (6)$$

This procedure can now be used to derive optimal plans for consumption in each period until we finally arrive in period 0.

To obtain optimal consumption plans one has to estimate Euler equations. Very often authors linearize nonlinear Euler equations using first or second order Taylor polynomials [1], [6], [7]. Linearized Euler equations were estimated using the general moment method and in many studies the estimated coefficients had opposite signs or were not stable [8], [3]. For described above framework, we do not calculate Euler equations analyticaly rather we find optimal consumption plans directly applying numerical methods. We obtaine that equilibrium consumption should be close to 62.6 mill. in 2008. As to the lastest data real private consumption stands at 58.6 mill., indicating that our estimation is slightly higher as the observed value.

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