Inexact Josephy–Newton framework for variational problems and its applications to optimization

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We propose and analyze a perturbed version of the classical Josephy-Newton method for solving generalized equations, and of the sequential quadratic programming method for optimization problems. This perturbed framework is convenient to treat in a unified way standard sequential quadratic programming, its stabilzed version [9, 2], sequential quadratically constrained quadratic programming [1, 4], and linearly constrained Lagrangian methods [8, 3].

For the linearly constrained Lagrangian methods, in particular, we obtain superlinear convergence under the second-order sufficient optimality condition and the strict Mangasarian–Fromovitz constraint qualification, while previous results in the literature assume (in addition to second-order sufficiency) the stronger linear independence constraint qualification as well as the strict complementarity condition.

For the sequential quadratically constrained quadratic programming methods, we prove primal-dual superlinear/quadratic convergence under the same assumptions as above, which also gives a new result.

Another possible application of the general framework is concerned with the development of truncated versions of sequential quadratic programming, which is regarded as a difficult issue for problems with inequality-type constraints [5].

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