On single machine due date assignment and scheduling with positional deterioration

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Single machine due date assignment (DDA) and scheduling problems are considered in which the processing time of each job is not constant but depends on its position in a processing sequence. The objective function to be minimized includes the cost of due date assignment, the total cost of discarded jobs and, possibly, the holding cost of the early jobs (total earliness). We mainly focus on scheduling models with a *deterioration* effect. Informally, under deterioration the processing time is not a constant but changes according to some rule, so that the later a job starts, the longer it takes to process.

If the jobs are processed in accordance with a certain permutation $\pi = (\pi(1), \pi(2), \ldots, \pi(n))$, then the processing time of job $j = \pi(r)$, i.e., of the job sequenced in the $r-$th position, is given by $p_j^{[r]} = p_j g(r)$, where $g(r)$ is a function (with $g(1) = 1$) that specifies a positional deterioration effect (if $g(r) \leq g(r+1)$ for each $r, 1 \leq r \leq n-1$), and p_j is the *normal* or *standard* processing time. Under positional *polynomial* deterioration, the actual processing time of a job j that is sequenced in position r is given by $p_j^{[r]} = p_j r^A$, where A is a given positive constant that is common for all jobs. Under positional *exponential* deterioration, the actual processing time of a job j that is sequenced in position r is given by $p_j^{[r]} = p_j \gamma^{r-1}$, where γ is a given constant, which is common for all jobs, representing a rate of deterioration if $\gamma > 1$. For the results on scheduling problems with positionally dependent processing times defined by polynomial functions, see Biskup (1999), Mosheiov (2001, 2005) and Gordon et al. (2008), and by exponential functions, see Wang (2005), Wang and Xia (2005) and Gordon et al. (2008).

In the problems under consideration, the jobs have to be split into two subsets denoted by N_E and N_T. The jobs of subset N_T are essentially discarded, and a penalty α_j is paid for a discarded job $j \in N_T$. In a feasible schedule only the jobs of subset N_E are sequenced, and each of these jobs is completed no later than its due date.

We consider two DDA models: (i) CON under which all jobs are given a common due date, and (ii) SLK under which the due date of a job is computed by increasing its actual processing time by a *slack* q, common to all jobs. The purpose is to select the due dates for the jobs and the sequence of the early jobs in such a way that a certain penalty function is minimized. We focus on two objective functions. One of them includes the cost of changing the due dates $\varphi(d)$ and the total penalty for discarding jobs, i.e., $F_1(d, \pi) = \varphi(d) + \sum_{j \in N_T} \alpha_j$, where π is the sequence of the early jobs, d is the vector of the assigned due dates. Another objective function additionally includes the total earliness of the scheduled jobs, i.e., $F_2(d, \pi) = \sum_{j \in N_E} E_j + \varphi(d) + \sum_{j \in N_T} \alpha_j$, where the earliness E_j of job j is the difference between its due date and its completion time.

For the CON model, $\varphi(d) = \beta d(\pi)$, where β is a positive constant and $d(\pi)$ is a common due date which depends on the sequence of early jobs. For the SLK model, $\varphi(d) = \beta q(\pi)$, where $q(\pi)$ is the slack which depends on the sequence of the early jobs.

We show that for any positional deterioration model (polynomial or exponential) in an optimal schedule the jobs are sequenced in LPT order of their normal processing times. We develop dynamic programming algorithms that minimize the functions F_1 and F_2 for deterioration mode of job processing times. The running times of our algorithms is $O(n^2)$ for the CON model and $O(n^3)$ for the SLK model. We also discuss how the obtained results can be extended to the models with a positional learning effect.

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