Parallel Newton methods for solving large-scale LP problems

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Many applied problems can be reduced to linear problems (LPs), for which numerical methods and software were developed long ago. It may seem that there is not difficult to find an optimal solution in such a problem. However, the dimension of modern problems (millions of variables and hundreds of thousands of constraints) is sometimes too large to be solved by conventional methods. Therefore, new approaches are needed for solving such problems on powerful computers.

In [1] a new method for solving LPs is proposed that is close to the quadratic penalty function method and to the modified Lagrangian function method. When applied to the dual LP, this method yields the exact projection of the given point on the set of solutions to the primal LP as a result of the one-time unconstrained maximization of an auxiliary piecewise quadratic concav function with a finite penalty coefficient. The use of the generalized Newton method [2] for the maximization of the auxiliary function enables one to solve LPs with a very large number of nonnegative variables (several tens of millions) and a moderate number of constraints (several thousand) on computers based on Pentium IV processors. This study is devoted to the parallelization of this method so as to make it possible to solve LPs with a larger number of constraints (up to several hundreds of thousands).

We propose several variants for parallelizing the generalized Newton method as applied to solving LPs [3]. We present some numerical results obtained on a parallel cluster. These results show that the proposed approach to solving LPs using the generalized Newton method can be efficiently parallelized and used to solve LPs with several million variables and up to two hundred thousand constraints. For example, for LPs with one million variables and 10000 constraints, one of the parallelizing schemes for 144 processors of the cluster MVC 6000IM accelerated the computations approximately by a factor of 50, and the computation time was 28 sec. A LP with two million variables and 200000 constraints was solved in about 40 min. on 80 processors. A LP with 60 million variables and 4000 constrained was solved in 140 sec. on 128 processors.

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