Close to regular plane covering by mobile sensors

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Let wireless sensor network (WSN) consists of the set J ($|J| = m$) of mobile sensors with adjustable sensing and communication ranges, and the sensors are distributed randomly over the plane region O of space S. Each sensor in *active* mode consumes its limited energy for sensing, communication and movement. In a *sleep* mode sensor preserves its energy. Suppose the sensing and communication areas of every sensor are the disks of certain radii with sensor in the center [1, 5, 7]. Region O is covered if every its point belongs to at least one sensing disk. The arc (i, j) belongs to the WSN, if sensor j is inside the communication range of sensor i. Lifetime of WSN is the number of time rounds during which region O is covered by connected active sensors. The problem is to maximize the lifetime of WSN. This problem is sufficiently complex, and even special cases are NP-hard [3]. Our goal is to compare the lifetime of WSN in the cases when the sensors are located in the certain places and form the regular covers [4, 5, 6, 7], and when the sensors are randomly distributed and do not move with two proposed types of covers by mobile sensors.

Let region O is tiled by regular triangles (*tiles*) with the side $R\sqrt{3}$. These triangles form a regular grid with the set of grid nodes I. Suppose each sensor has the energy storage $q > 0$. For any sensor, sensing energy consumption per time round depends on a sensing range r and equals $SE = \mu_1 r^a,\ \mu_1 > 0,\ a \geq 2$; communication energy consumption per time round depends on the distance d and equals $CE = \mu_2 d^b,\ \mu_2 > 0,\ b \geq 2$; and the energy consumption per time round during the motion depends on the speed v and equals $ME = \mu_3 v^c,\ \mu_3 > 0,\ c > 0$. We suppose that during the motion, sensor do not consumes the energy for sensing and communication.

If all sensors have the same sensing ranges R, and they are equally placed in the grid nodes, then the regular cover, we call it $A1$, [2, 4, 6] is optimal with respect to the sensing energy consumption (or covering density). In the cover $A1$ each triad of neighbor disks of radius R with centers in the nodes of triangle has one common point in the center of the tile. Every sensor, located in the node i, must cover the disk of radius R with center in the node i (we call it *disk* i). The density of this cover is $D_{A1} = 2\pi/\sqrt{27} \approx 1.2091$ [2, 4], and the sensing energy consumption of every sensor equals $SE_{A1} = \mu_1 R^a$. Since communication distance for each sensor equals $R\sqrt{3}$, then the communication energy consumption is $CE_{A1} = \mu_2(R\sqrt{3})^b$. Therefore, the lifetime of one sensor equals $t_{A1} = q/(\mu_1 R^a + \mu_2(R\sqrt{3})^b)$.

Since the minimal number of grid nodes is $N \approx 2S/(R^2\sqrt{27})$ [4], then the lifetime of cover $A1$ is bounded by the

$$L_{A1} \approx \frac{mt_{A1}}{N} \approx \frac{mq\sqrt{27}}{2S(\mu_1 R^{a-2} + \mu_2 R^{b-2}(\sqrt{3})^b)}.$$

Let the sensors are distributed uniformly over the region O, and parameter $a_{ij} = 1$ if i is the closest grid node to the sensor j (i.e. the distance between j and i is $d_{ij} = \min_{k\in I} d_{kj}$), and $a_{ij} = 0$ otherwise. Denote the set $J_i = \{j \in J | a_{ij} = 1\}$. Then the sensors inside the regular *hexagon* i with center in the node i and the sides at the distance $\delta = R\sqrt{3}/2$ from the center, are in the set J_i. We reasonably suppose that the sensor j in J_i (or in the hexagon i) must cover the disk i. Then if j is located on the distance r away from the grid node i, then it must increase the sensing range by r in order to cover the disk i. Moreover, if the distance between the node i and sensor $j_1 \in J_i$ is r_1, and the distance between the node k and sensor $j_2 \in J_k$ is r_2, then in order to guarantee a communication between the neighbor sensors j_1 and j_2 it is necessary to increase the communication range of j_1 and j_2 by at least $r_1 + r_2$. Additionally every mobile sensor $j \in J_i$ could move towards the node i during some time rounds in order to be nearer i. For the sake of simplicity, we suppose that the speed of every sensor possess the two values 0 or v. Therefore, if sensor $j \in J_i$ is moving, then the speed v and direction (towards the grid node i) are known.

Let us consider the concentric circles of radii $\delta_k = k \cdot v,\ k = 1, \ldots, K = [\delta/v]$. Denote the set $J_i^k = \{j \in J_i | \delta_{k-1} < d_{ij} \leq \delta_k\}$. Then any sensor $j \in J_i^k$ could reach the node i by at most k time rounds. Since the resource of each sensor is limited by q, then if any sensor $j \in J_i^k$ moves l time rounds and, as a result, consumes $l\mu_3 v^c$ units of its energy, then taking into account the remainder sensor-node distance $(k-l)v$, the sensor can be active during

$$t_k(l) \approx \frac{q - l\mu_3 v^c}{\mu_1(R+(k-l)v)^a + \mu_2(R\sqrt{3}+2(k-l)v)^b}$$

time rounds. Function $t_k(l)$ is concave, then one can find $T_k = t_k(l_k) = \max_{0\le l\le k} t_k(l)$ by $O(\log_2 K)$ complexity.

Since the sensors are distributed uniformly, then there are $N_k \approx m\pi(2k-1)v^2/S$ sensors in every set J_i^k. Let first active sensors are initially located in J_i^1, and we suppose that they do not move and are active during

$$L_1 = \frac{qN_1}{\mu_1(R+v)^a + \mu_2(R\sqrt{3}+2v)^b}$$

time rounds. During L_1 time rounds the sensors in J_i^2 could move towards the grid node i, and then they can be active during $L_2 = N_2 \max_{0\le l\le \min\{2,L_1\}} t_2(l)$ time rounds. Therefore, during $\Lambda_{k-1} = \sum_{l=1}^{k-1} L_l$ time rounds the sensors in J_i^k could move to the grid node i, and then they can be active during $L_k = N_k \max_{0\le l\le \min\{k,\Lambda_{k-1}\}} t_k(l)$ time rounds. As a result, we get the estimation of the WSN's lifetime as $\Lambda^\delta = \sum_{k=1}^{K} L_k$.

The above results depend on the parameter δ and obtained in case of fixed grid. Suppose the number of sensors N_k in J_i^k is sufficiently great for each $k = 1, \ldots, K = [\delta/v]$. If grid is wandering, then we may replace it several times without change the size (the new grid node i_n is relocated from the previous position i_{n-1} by 2δ distance), then the WSN's lifetime can be increased as follows. Let us set $\delta = v$ and suppose that during the first time round, when a portion of sensors in $J_{i_1}^1$ are active, other sensors in every $J_{i_n}^1,\ n \ge 1$ move to the grid node i_n. The number of sensors in each $J_{i_1}^1$, which are active during the first time round, is $n'_1 \approx (\mu_1(R+v)^a + \mu_2(R\sqrt{3}+2v)^b)/q$, and we suppose that $n'_1 \le N_1$. These sensors do not move and must increase their sensing ranges by v. During the first time round $N_1 - n'_1$ sensors in each set $J_{i_1}^1$ will reach the grid node i_1, and it is not necessary to increase their sensing ranges to cover the O. Moreover, each sensor in $J_{i_n}^1,\ n \ge 2$, will reach i_n during the first time round. The number of sensors in every set $J_{i_n}^1,\ n \ge 2$, is N_1, and the number of these sets (new grid nodes) is $n'_2 \ge [R\sqrt{3}/(2v)]^2 - 1$ (value n'_2+1 is the number of disks of radius δ packed in $R\sqrt{3}$-side rhomb). Every sensor, located outside the sets $J_{i_n}^1,\ n \ge 1$, has two time rounds to reach the nearest grid node. The number of such sensors in the rhomb is

$$n'_3 \approx \frac{R^2 m\sqrt{27}}{2S} - (n'_2+1)N_1 \ge 0.24\frac{R^2 m}{S}.$$

Since $N_1 \approx m\pi v^2/S$, then the WSN's lifetime in this case equals

$$\Lambda^v \approx 1 + (N_1 - n'_1 + N_1 n'_2)\frac{q-\mu_3 v^c}{\mu_1 R^a + \mu_2(R\sqrt{3})^b} + n'_3\frac{q-2\mu_3 v^c}{\mu_1 R^a + \mu_2(R\sqrt{3})^b}.$$

The above results depend on the radius R which, in turn, determines the tile's size. The lifetime L_{A1} of model $A1$ is irrespective R. If the sensors are distributed accidentally, then the WSN's lifetime is less than L_{A1}, and in this paper we compare the values Λ^δ and Λ^v with the upper bound L_{A1}. Moreover, we've found the cases when $\Lambda^\delta \ge \Lambda^v$ or $\Lambda^\delta < \Lambda^v$ and show that almost in all cases mobile sensors gives a considerable gain in WSN's lifetime in comparison with the static sensors case.

References

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