A multiobjective synthesis of optimal control system by the network operator method

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The following problem of synthesis of control is considered.

The system of differential equations which describes the dynamics of the object is given

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}),$$

where $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{u} \in U \subset \mathbf{R}^m$, U is closed bounded domain.

The domain of existence of initial values is given

$$\mathbf{x}(0) = \mathbf{x}^0,\ \mathbf{x}^0 \in X_0,\ X_0 \subset \mathbf{R}^n.$$

It is necessary to find the control in the following form

$$\mathbf{u} = \mathbf{g}(\mathbf{x})$$

to satisfy boundary conditions $\mathbf{u} \in U$ and minimize the functionals

$$J_i = \int \dots \int \int_0^{t_f} f_0(\mathbf{x}, \mathbf{u}) dx_1^0 \dots dx_n^0 dt,$$

where the multiple integral is calculated on domain X_0, t_f is the given time, $i = \overline{1, l}$.

Pareto set is considered to be the solution of problem

$$\tilde{P} = \{\tilde{\mathbf{g}}^i(\mathbf{x}) : i = \overline{1, K}\}.$$

For the Pareto set the following condition is right $\forall \mathbf{g}(\mathbf{x}) \in U\ \exists \tilde{\mathbf{g}}^i(\mathbf{x}) \in \tilde{P}$ such that $\mathbf{J}(\tilde{\mathbf{g}}^i(\mathbf{x})) \leq \mathbf{J}(\mathbf{g}(\mathbf{x}))$, where $\mathbf{J}(\mathbf{g}(\mathbf{x})) = [J_1(\mathbf{g}(\mathbf{x})) \dots J_l(\mathbf{g}(\mathbf{x}))]^T$.

Theorem 1 *Assume X_0 is a finite denumerable set*

$$X_0 = \{\mathbf{x}^{0,1}, \mathbf{x}^{0,2}, \dots, \mathbf{x}^{0,M}\}.$$

Suppose $\tilde{P}$ is the solution of the multiobjective synthesis problem and $\hat{P}$ is the solution of the same problem but with the set of initial conditions $\hat{X}_0 \subseteq X_0$. Then $\hat{P} \subseteq \tilde{P}$.

From the theorem 1 we can make conclusion. Suppose $\hat{X}_0 = \{\hat{\mathbf{x}}^0\}$ and $\mathbf{u} = \hat{\mathbf{g}}(\mathbf{x})$ is the solution of this problem. Then $\exists \tilde{\mathbf{g}}(\mathbf{x}) \in \tilde{P}$ such that $\mathbf{J}(\tilde{\mathbf{g}}(\mathbf{x})) \leq \mathbf{J}(\hat{\mathbf{g}}(\mathbf{x}))$.

For numerical solution of the problem of multiobjective synthesis it is necessary to create the space of different functions including piecewise continuous and not continuously differentiable ones. From this space we take functions that belong to Pareto set. Such space of functions can be created with the help of network operator[1].

Mathematical equation consists of variables, parameters, unary and binary operations that form four constructive sets.

Set of variables $X = (x_1, \dots, x_n)$.

Set of parameters $Q = (q_1, \dots, q_p)$.

Set of unary operations $O_1 = (\rho_1(z), \dots, \rho_W(z))$.

Set of binary operations
$O_2 = (\chi_0(z', z''), \dots, \chi_V(z', z''))$.

Unary operations set must have an identity operation $\rho_1(z) = z$. Binary operations must be commutative $\chi_i(z', z'') = \chi_i(z'', z')$, associative $\chi_i(z', \chi_i(z'', z''')) = \chi_i(\chi_i(z', z''), z''')$ and have unit element $\forall \chi_i(z', z'') \in O_2\ \exists e_i\ \chi_i(z', e_i) = z'$, $i = \overline{0, V}$.

Definition 1 *Program notation of mathematical equation is a notation of equation with the help of elements of constructive sets X, Q, O_1, O_2.*

Definition 2 *Graphic notation of mathematical equations is the notation of program notation that fulfills the following conditions:*

a) binary operation can have unary operations or unit element of this binary operation as its arguments;

b) unary operation can have binary operation, parameter or variable as its argument;

c) binary operation cannot have unary operations with equal constants or variables as its arguments.

Theorem 2 *Any program notation can be transformed in graphic notation.*

We use graphic notation to construct a graph of mathematical equation. Source nodes of the graph accordings to elements of the sets of variables or parameters. Other nodes of the graph accordings to the binary operations. Edges of the graph link with the unary operations.

Definition 3 *Network operator is a directed graph with following properties:*

a) graph should be circuit-free;

b) there should be at least one edge from the source node to any nonsource node;

c) there should be at least one edge from any nonsource node to sink node;

d) every source node corresponds to the item of set of variables X or parameters Q ;

e) every nonsource node corresponds to the item of binary operations set O_2 ;

f) every edge corresponds to the item of unary operations set O_1 .

To present the network operator in a computer we use an integer matrix. Let us numerate all nodes of network operator so that the number of node from which the edge comes out is smaller than the number of the node this edge comes in. Such numeration can always be done for oriented circuit-free graph.

Definition 4 *Network operator matrix (NOM) is an integer upper-triangular matrix that has as its diagonal elements numbers of binary operations and nondiagonal elements are zeros or numbers of unary operations, besides if we replace diagonal elements with zeros and nonzero nondiagonal elements with ones we shall get an vertex incident matrix of the graph that satisfies conditions a-c of network operator definition.*

To calculate the mathematical equation with the help of NOM it is necessary to introduce three integer vectors:
- vector of numbers of nodes for variables $\mathbf{b} = [b_1 \dots b_n]^T$, where b_i is the number of source node in the network operator that is linked to variable x_i;
-vector of numbers of nodes for parameters $\mathbf{s} = [s_1 \dots s_p]^T$, where s_i is the number of the source node in the network operator that is linked to parameter q_i;
- vector of numbers for output variables $\mathbf{d} = [d_1 \dots d_m]^T$, where d_i is the number of the node in the network operator that is linked to output variable u_i.

Theorem 3 *If we have the NOM $\Psi = [\psi_{ij}]$, $i, j = \overline{1, L}$, and vectors of numbers of nodes for variables $\mathbf{b} = [b_1 \dots b_n]^T$, parameters $\mathbf{s} = [s_1 \dots s_p]^T$ and outputs $\mathbf{d} = [d_1 \dots d_m]^T$ then it is sufficient to calculate the mathematical expression described by NOM.*

To store intermediate results let us introduce additional vector of nodes $\mathbf{z} = [z_1 \dots z_L]^T$ is introduced.

At first we set initial values for the vector of nodes.

$$z_i^0 = \begin{cases} x_k, \text{ if } i = b_k \\ q_j, \text{ if } i = s_j \\ e_{\psi_{ii}}, \text{ if node } i \text{ is not source} \end{cases} .$$

Then we calculate according to the following

$$z_i^j = \begin{cases} \chi_{\psi_{jj}}(z_j^{i-1}, \rho(z_i^{i-1})), \text{ if } \psi_{ij} \neq 0 \\ z_j^{i-1}, \text{ otherwise} \end{cases} .$$

To find the solution of the synthesis problem we use the genetic algorithm [2, 3]. We generate population of NOMs and search in it solutions of the Pareto set. At the construction of the genetic algorithm we use method of basic solution variation.

For network operator the following variations are defined:

0) replacement of unary operation on the edge;
1) replacement of binary operation in the node;
2) addition of an edge with a unary operation;
3) addition of a node with a binary operation;

All variations on the network operator can be presented as an integer variation vector that consists of four elements: $\mathbf{w} = [w_1\, w_2\, w_3\, w_4]^T$, where w_1 is the number of variation, w_2 is the number of row in NOM,w_3 is the number of column in NOM, w_4 is the number of unary or binary operation.

To construct a set of mathematical equations we define one basis mathematical equation that is described by basic network operator and set of variation vectors Other mathematical expression and its network operator Ψ^i is obtained as a result of variation of basic solution Ψ^0.

$$\Psi^i = \mathbf{w}^i \circ \Psi^0.$$

References

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