APPLICATION OF CONTROL IN ORDINARY DIFFERENTIAL EQUATION SOLVERS*

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Numerous papers [1]–[3] show that the attention towards nonconventional methods for numerical solution of ordinary differential equations rapidly grows nowadays. First of all, this is related lo the theory and methods for numerical solutions of problems with large parameters. Let us cite the main reasons which result in an extremal situation (i.e., appearance of a large parameter) in a Cauchy problem. Those are: (i) the high accuracy in solving the Cauchy problem; (ii) a large integration interval; (iii) a high order of the system; (iv) too many points at which the right-hand side is nonsmooth; (v) system stiffness, etc.

Implicit and special schemes are used to integrate systems in extremal situations. It should be noted that when using implicit schemes, it is difficult to solve nonlinear algebraic equations arisen, because the Jacobi matrix is ill-conditioned. The properties of systems to integrate can be "improved" in some cases by norm ing and changing variables.

Note that various definitions of stiffness are employed in work on numerical methods. For example, the concept of stiffness in [2] includes ideas of the theory' of singularly perturbed equations. Other authors classify systems as stiff if the large parameter is the ratio of the largest and least absolute values of the eigenvalues of the Jacobi matrix. All the presented definitions can be considered in the framework of problems with large parameters [3]. It should be noted that methods for solving problem with large parameters are closely connected with regularization methods for ill-conditioned systems (in particular, with solution of nonlinear systems and expansion with respect to the parameter).

In the present paper we introduce controlling parameters and propose cost-effective explicit methods for numerical solution of stiff ordinary differential equations. The presented methods are also used as the first approximation in implicit schemes. Most numerical methods include parameters and variables that need to be adjusted during the execution. The shortcomings of the classical approach and some difficulties of the stiff systems methodology can be remedied for ill-conditioned systems by using proven techniques from optimal control theory with state and control-state constraints. For a example, singular perturbation problems form a special class of problems containing a parameter ε. When this parameter is small, the corresponding differential equation is stiff; when ϵ tends to zero, the differential equation becomes differential algebraic. Usually, the limit case $\varepsilon = 0$ much easier to analyze.

For explicit and implicit methods we introduce control variables: type, stage, coefficients, choice of step size. The cost function – local and global errors. Many systems of differential equations have known invariants, that is, relations that hold along any solution trajectory. Numerical methods do not always maintain the constancy of these invariants; but it is often desirable to conserve them because the solution of the equation might be quite sensitive to small changes in their value. Hamiltonian (Pontiyagin) function is a common example of a invariant in optimal control theory and one serves as verification of the numerical results.

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