Equilibrium Model of Credit Market

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Equilibrium credit model present a system of optimization problems describing the behavior of two macroscopic players: a borrow and a creditor. The decision making process for either is described by convex programming problem, and the decisions made are balanced using variational inequalities or, equivalently, linear optimization problems. The model has the form

$$w^* \in \text{Argmax}\{S_1(w) + (1 + r^*)M(w) \ \ | \\ g(w) \le y^*, \ \ w \in W\}, \quad (1)$$

$$\langle p - p^*, g(w^*) - y^* \rangle \le 0, \ p \ge 0, \quad (2)$$

$$y^* \in \text{Argmax}\{S_2(y) + \langle p^*, y\rangle \ \ | \\ \langle m, y\rangle \le M(w^*), y \in Y\}. \quad (3)$$

$$(r - r^*)(\langle m, y^*\rangle - M(w^*)) \le 0, \ r \ge 0. \quad (4)$$

Here, $S_1(w), S_2(y), M(w)$ are upward convex (concave) functions for $w \in W_0$ and $y \in Y$, where $W_0 \subset R^n$ and $Y \subset R^m$ are convex closed sets. The first two functions describe the expected profit of the agents; the last function describes the borrowed funds in the form of credit; $g(w)$ is a downward convex function for forming balances in the production sector; and r^* is a interest rate or the credit price, which is the basic model parameter determining the unsaturated demand and, accordingly, the equilibrium of the system.

Problems (1) and (3) describe the decision making processes for the borrower and the creditor. The primal solutions found, namely, w^* and y^*, are used to form balances of each other. They are contained in balances (2) and (4). Moreover, in the course of the decision making, the agents make dual decisions, namely, $p^* \in C$ and $r^* \in R^1_+$, where R^m_+ and R^1_+ are positive orthants. Using this information, they influence the each other's objective functions, there, the decision making process. The vector $p^* \ge 0$ forms objective function (3), while the number $r^* > 0$ (according to its interpretation) must not degenerate to zero, since the objective function of problem (3) is assumed to be unsaturated. This means that the maximum of problem always lies on the boundary of the set and r^*, as Lagrange multiplier, is nonzero. The fixed vector $m > 0$ is the market price vector, which is used to buy the resource vector $y^* \in Y$.

Problem (3) partitions $M(w^*) > 0$ into the sum $x_1 + ... + x_k \le M$. In particular, the partition can be exact. Then, $x_1 + ... + x_k = M$. The distribution and redistribution of resources (primarily financial ones) is a central problem in any economic activity. In the model, some financial resources are allocated for the purchase of material resources to support technologies associated with production. In this case, $x_i = my_i$, where y_i is a unit of some resource and m_i is its price in monetary units.

Note that the two agents interact according to the following scheme. The creditor grants the resource vector y^* and the interest rate $r^* > 0$ to the borrower, while the borrower gives the creditor the vector of Lagrange multipliwers (internal price) p^*, which is used by the creditor to form the profit function for the participation in the project and to find the required credit $M(w^*)$. In this scheme, the dual vector p^* and R^* play the role of feedback ensuring an equilibrium state of system (1)-(4).

To solve system (1)-(4) we use the duel extraproximal method

$$\bar{p}^n = \pi_+\{p^n + \alpha(g(w^n) - y^n)\},$$

$$\bar{r}^n = \pi_+\{r^n + \alpha(\langle m, y^n\rangle - M(w^n))\},$$

$$w^{n+1} \in \text{argmin}\{\frac{1}{2}|w - w^n|^2 + \\ +\alpha(S_1(w) - (1 + \bar{r}^n)M(w)) + \langle \bar{p}^n, g(w)\rangle) \ \ | \ \ w \in W\},$$

$$y^{n+1} \in \text{argmin}\{\frac{1}{2}|y - y^n|^2 + \\ +\alpha(S_2(y) - \langle \bar{p}^n - \bar{r}^n m, y\rangle) \ \ | \ \ y \in Y\},$$

$$p^{n+1} = \pi_+\{p^n + \alpha(g(w^{n+1}) - y^{n+1})\},$$

$$r^{n+1} = \pi_+\{r^n + \alpha(\langle m, y^{n+1}\rangle - M(w^{n+1}))\}.$$

Monotonical in the norm convergence of this process is proved, if parameter subject to condition $0 < \alpha < \alpha_0$.