On the influence of cost functional on the solution of the optimal control problem of the crystallization process

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The optimal control problem of metal solidification in casting is considered. For metal crystallization a special device is used. It consists of upper and lower parts. The upper part consists of a furnace with a mold moving inside. The lower part is a cooling bath consisting of a large tank filled with liquid aluminum whose temperature is somewhat higher than the aluminum melting point. The cooling of liquid metal in the furnace proceeds as follows. On the one hand, the mold is slowly immersed in the low-temperature liquid aluminum, which causes the solidification of the metal. On the other hand, the mold gains heat from the walls of the furnace, which prevents the solidification process from proceeding too fast. So different parts of the mould external bounder are in different heat conditions, depending upon time. The crystallization process is affected by different phenomena such as heat losses due to its radiation, obtaining of energy by the mould due to expanse of radiation from aluminum and furnace, heat exchange between liquid aluminum and the mould. The complication of the task is that the metal can be present at considered conditions simultaneously in two phases: solid and liquid.

The process of solidification in metal is modelled by a three-dimensional, two-phase, initialboundary value problem of the Stefan type. A numerical algorithm is presented for solving the initialboundary value problem [1]. Primary attention was given to the evolution of the solidification front. The evolution of the solidification front is affected by numerous parameters. The influence of the objects velocity on the solidification front is of special interest in practice.

The optimal control problem is to choose a regime of metal cooling and solidification at which the solidification front has a preset or nearly preset shape (namely, a plane orthogonal to the vertical axis of the object) and moves sufficiently slowly (at a speed close to the preset one). The time-dependent speed of movement of the mould along the furnace was selected as the control.

Several studies connected with the selection of a cost functional, which models technological requirements for the process of crystallization in the formulated optimal control problem, were carried out. The first functional is the time average standard deviation of the real interface between two phases (liquid and solid) from the desired one. The desired interface was a plane that moved with an assigned constant velocity. This functional was used to ensure fulfilling the requirements, superimposed both on the shape of the real interface, and on the speed of its displacement. As numerous calculations have shown, the first functional ensures a speed that is close to the desired one and provides some straightening of the interface.

Another functional was proposed to provide a higher degree of straightening. This second functional was the time average standard deviation of the real interface from a plane located in the middle of the real interface. The calculations have shown that the real interface in this case becomes noticeably nearer to a plane.

The optimal control problem was solved numerically using the gradient method. The gradient of the cost function was found with the help of conjugate problem. The discreet conjugate problem was posed with the help of Fast Automatic Differentiation technique [2].

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References

[1] A. F. Albu and V. I. Zubov, *Mathematical modeling and study of the process of solidification in metal casting.* Zh. Vychisl. Mat. Mat. Fiz., 2007, V.47, No. 5. [Comp. Math. Math. Phys., 2007, V.47].

[2] A. F. Albu and V. I. Zubov, *Calculation of the gradient of functional in one optimal control problem.* Zh. Vychisl. Mat. Mat. Fiz., 2009, V.49, No. 1. [Comp. Math. Math. Phys., 2009, V.49].