Parallel programming in MATLAB for modeling an economy

Vasily Dikusar [1], Nicholas Olenev [2]

[1] Dorodnitsyn Computing Center at FRC "Computer Science and Control" of RAS
Vavilov st. 40, 119333 Moscow, Russia
dikussar@yandex.ru

[2] Dorodnitsyn Computing Center at FRC "Computer Science and Control" of RAS
Vavilov st. 40, 119333 Moscow, Russia
nolenev@mail.ru

Abstract. *We consider here the use of MATLAB parallel applications for mathematical modeling of the economy on example of the identification problem of economic models. By identification of a Ramsey type model of the economy two possible scenarios for the development of the Russian economy are considered.*

1 Introduction

Message passing interface (MPI) is complex enough but serves as standard de-facto for parallel calculations [1]. So, companies - developers of mathematical packages - become themselves to implement applications for parallel computing in their packages. For example, MathWorks has developed an application for the creation of parallel and distributed programs using the MPI library funds and their implementation on the platform of MATLAB [2], which simplifies the practical use of parallel computing on multicore computers, clusters, and GRID-systems.

To identify the external parameters of dynamic models of economic systems they have developed specific criteria of proximity and similarity to statistical and calculated by model time series for macro indexes of the economic system [3].

The problem of model identifying in an indirect way is formulated as an optimal control problem with constraints in the form of differential equations.

To determine the optimal geometry of the trajectories of the initial problem is reduced to a discrete and solved by methods of linear and non-linear programming. The boundary value problem is solved with the use of continuous analogue of the Newton and gradient methods. The Jacobian matrix is calculated using parallel procedures. To solve the problems of linear and non-linear programming methods are used factor analysis and the results are clear schemes for stiff systems [4].

It solved the problem of eigenvalues for the matrix of observations in the case of linear large-scale problems. Based on MATLAB software package designed for problems of an optimal control with the use of distributed computing and GRID-technologies for the numerical solution of this problem of eigenvalues.

For the administration and configuration of parallel calculations in MATLAB, they use two applications:

(1) Parallel Computing Toolbox (PCT),

(2) MATLAB Distributed Computing Server (MDCS).

To check the availability of these applications in MATLAB the command you must be run is:

>>ver

You can develop your program on a multicore desktop computer using Parallel Computing Toolbox and then scale up it to use a cluster supercomputer, a cloud, or a grid by running it on MATLAB Distributed Computing Server [2].

2 Model description

Let us consider a Ramsey type dynamic model of economy [2, 5, 6]. Let's gross domestic product (GDP) at constant 2000 prices $Y(t)$ is determined by homogeneous production function with constant elasticity of substitution (CES-function).

$$Y(t) = Y_0 \left[a \left(\frac{L}{L_0} \right)^{-b} + (1-a) \left(\frac{K}{K_0} \right)^{-b} \right]^{-1/b}, \tag{1}$$

where $Y_0, L_0, K_0, a \in (0,1), b > -1$ are constant parameters.

Typically, they determine the parameters of production function directly by the economic statistics for time series of variables involved in the production function $Y(t), L(t), K(t)$. But sometimes it is not possible. For example in Russia 2000-2006 GDP grew up, but employment and capital remained practically unchanged [2]. In addition, the value of the capital represented by the statistics is in serious doubt [2]. Only capital involved in the production process affects on output of GDP, it has objective value a certain "effective" capital which is expressed in constant prices of 2000, and here we will try to evaluate it.

Labor measured in this model by the average annual number of employees in the national economy is growing at a constant rate $\gamma > 0$

$$dL/dt = \gamma L(t), L(0) = L_0. \tag{2}$$

Capital changes due to the usual equation

$$dK/dt = J(t) - \mu K(t), K(0) = K_0, \tag{3}$$

where μ is a depreciation rate. Suppose that every time product balance in constant prices of 2000 is performed.

$$Y(t) + \pi_I(t)I(t) = Q(t) + \pi_J(t)J(t) + \pi_E(t)E(t), \tag{4}$$

where $I(t)$ is imports, $J(t)$ – investments, $E(t)$ - exports, $Q(t)$ – total consumption of households, government, and public organizations in prices of GDP; $\pi_I(t)$ is relative price index on import, $\pi_J(t)$ – on investment, and $\pi_E(t)$ – on export.

Table 1: Statistical data for Russian economy 2000-2006

Index	2000	2001	2002	2003	2004	2005	2006
$L(t)$	65.273	65.124	66.358	67.247	67.244	68.719	69.600
$\pi_E(t)$	1	0.84442	0.76610	0.72863	0.68475	0.69651	0.67010
$\pi_I(t)$	1	0.89204	0.82339	0.73075	0.59196	0.52193	0.45556
$\pi_J(t)$	1	1.02043	1.00752	0.97393	0.93350	0.88821	0.85997
$Y(t)$	7305.6	7676.9	8039.3	8625.8	9268.8	9817.6	10478.0
$I(t)$	1755.8	2084.1	2388.4	2811.2	3466.2	4055.4	4878.3
$J(t)$	1165.2	1265.7	1300.0	1462.2	1633.6	1807.2	2051.7
$E(t)$	3218.9	3354.1	3699.6	4162.0	4653.1	4950.9	5297.5
$Q(t)$	4677.3	5412.2	5861.9	6223.4	6609.5	6880.7	7386.3

Source: Russian Federation Federal State Statistis Service www.gks.ru [2].

3 Identification of the model

Sometimes you can find econometric dependence of exports, imports and investments from GDP. For example, [2] finds three constants δ, ρ, σ for Russia 2000-2006:

$$\begin{aligned}
&\pi_E(t)E(t) = \delta Y(t),\\
&\pi_I(t)I(t) = \rho(Y(t) - \pi_E E(t)),\\
&\pi_J(t)J(t) = \sigma(Y(t) + \pi_I I(t)).
\end{aligned}$$

To identify the model you should be set external parameters: three relative prices $\pi_E(t), \pi_I(t), \pi_J(t)$, seven parameters $a, b, \gamma, \mu, \delta, \rho, \sigma$ and three initial values, so that time series of macroeconomic indexes calculated by the model were close to statistical time series of them. Statistical data for Russian economy of 2000-2006 are shown in Table 1. The components of GDP are given in constant prices of 2000, bln. Rub. The relative price indices are obtained by calculation on the basis of data on changes in components of GDP at current and constant prices and the implicit GDP deflator. The values $Q(t)$ are determined by (4).

In contrast to [2] we use here a new index of similarity for two time series, which based on a modification of one of the Theil inequality index. If $X(t)$ is time series of a macroindex calculated by model, and $X_S(t)$ is its statistical time series then new index of simularity $U(X, X_S)$ is defined as

$$U(X, X_S) = \sum_{t=1}^{T} X(t)\,(2X_S(t) - X(t))\,/\sum_{t=1}^{T} X_S(t)^2, \tag{5}$$

where T is the number of years in time series.

The identification problem [7] is formulated as the next one optimal control problem [8]. It is necessary to find the maximum for convolution of criteria.

$$U(Y, Y_S)U(Q, Q_S)U(I, I_S)U(E, E_S)U(J, J_S) -> \max_{a,b,\gamma,\mu,\delta,\rho,\sigma,K_0} \tag{6}$$

under conditions (1)-(4).

Problem of parameter identification are solved for several countries using MATLAB [9]. The identified models are used for scenario calculations.

4 Some results

Results of the model identification are presented graphically in Fig. 1–3.

Figure 1: Results of the model identification for capital K and output Y.

Figure 2: Results of the model identification for import I and export E.

Figure 3: Results of the model identification for real investment J and final consumption Q.

Numerical results of identification [2]: $a = 0.84$, $b = 0.78$, $\mu = -0.175$, $\alpha = 0.41$, $K_0 = 17819$ bln. 2000. A negative value of μ means that the effective capital grows faster than it provides the real investment. So, the old capital of the Soviet period is involved in the production. But the value of old capital was not unlimited. You can estimate the time of its exhaustion. Assume that the maximum amount of the old capital, which can be involved without investing, is four times greter then the volume of the effective capital in 2000, so a time T of exhaustion of the old capital was estimated at the end of 2006 as $T = 8$ [2]. So at the end of 2008 the volume of involving old capital should be exhausted.

Therefore, the baseline scenario is pessimistic. See Fig. 4–9.

The method of model identification presented here was used in [9] for some other countries.

5 Conclusions

The model makes it impossible to clearly answer the question under what scenario the Russian economy will develop . One can only conclude that further development depends essentially on economic policy which will be held in the near future. If you do not take any effort the increase due to the involvement of the old capital will stop soon. If the economic policy will be based on the use of scientific and technological progress it is possible a good prognosis.

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Figure 4: Labor L in the baseline scenario of the forecast.

Figure 5: Capital K in the baseline scenario of the forecast.

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Figure 6: GDP Y in the baseline scenario of the forecast.

Figure 7: Export E in the baseline scenario of the forecast.

Figure 8: Import I in the baseline scenario of the forecast.

Figure 9: Real investment J in the baseline scenario of the forecast.