Optimal investment problems in a dynamic model of economy with venture capital

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The paper is denoted to the investigation of the schemes of investing in innovative projects. The work started to build a dynamic model of economy based on venture capital [1]. Seven economic agents are allocated: large producers, small producers, Government, venture capitalists, banking system, households, and trade intermediates. It is supposed that small enterprises exist by the venture capital. A dynamic model of a lifecycle during an investment period of such a firms is given [2].

The optimal control problem for a small firm can be formulated in the following way:

$$k(T) \longrightarrow \max_{y,w},\tag{1}$$

$$\dot{m} = -\mu m + I,$$

$$\dot{k} = -\beta k + bI,$$

$$\Pi_{u} := (p_{1} - se^{\mu t}v - ap)y = w + pbI,$$
(2)

$$(1)$$

$$(2)$$

$$(3) k(0) = bm(0) = \Lambda,$$

$$y \in [0, m],$$

$$W(T) := \int_{0}^{T} \frac{w}{p} dt + \gamma k(T) \ge \alpha \Lambda.$$

$$(3)$$

Where k – is a capital of a firm, m – its capacity (maximum output per unit of time), y – its real output, μ and β – rates of disposal for capacity and capital respectively, $\beta > \mu$. Parameters p and p_1 – prices

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for the products of large producers and innovative firms respectively. A is an amount of initial investments, I – own investments, w – payments for the initial investments. Investor's expected minimal rate of return is $\alpha > 1$, the interest of the investor in the capital of the firm is $\gamma \in [0, 1]$. T – is a duration of the investment period, parameters a and b are ratio of direct costs and incremental capital intensity, which shows an amount of a product consumed per creation a unit of capacity. v – nominal labor input, $\lambda = e^{\mu t}v$ – real labor input.

In this problem y and w are the control parameters. They are linked by the restriction (2), which defines the profit function Π_y of a firm. W(T)- is a total payment for the investments, inequality (3) defines a specific scheme of payments. The optimal control problems, corresponding to different patterns of investment and payments are solved. In the first scheme it is assumed the withdrawal period of the capital during which the investor receives everything he is supposed to, in the second it is assumed that a part of the income the investor receives as a fixed share in the capital of the company at the end of the investment period.

In the given model the following statements are true.

Statement 1. In the absence of bank credits it is optimal for small firms to make payments for the investments at the end of a period.

Statement 2. (1) gains its maximum on the same optimal control as a total discounted profit does, so it can be replaced by

$$\int_{0}^{T} \Pi_{y} e^{-\delta t} dt \longrightarrow \max_{y,w}$$

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