About features of differential models for a bank

Vasily Dikusar[1], **Nicholas Olenev**[1]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;*
dikussar@yandex.ru, nolenev@yahoo.com

Problems with a small parameter in the derivative refer to stiff equations [1-5]. It is difficult to formulate a precise definition of stiffness, but the main idea is that the equation includes some terms that can lead to rapid variation in the solution.

In applying explicit methods to solve stiff problems step size is limited to numerical stability rather than accuracy. Very strong stability of the differential equation is a disadvantage in terms of the accuracy of the numerical solution using the classical explicit methods [1-5].

As an example let consider the linear problem in general form:

$$\dot{y}(t) = Ay(t) + g(t), g \in R^n, A \in R^{n \times n}, y \in R^n, y(0) = y_0, \qquad (1)$$

where A – is a constant value matrix. Let $\lambda_1, \lambda_2, ..., \lambda_n$ are eigenvalues of the matrix A. The problem (1) is called stiff if

1. There are λ_i for which $Re\lambda_i \ll 0$.

2. There are λ_i such that they are small in comparison with the absolute value of the eigenvalues satisfying item 1.

3. There is no λ_i with a large positive value of real part.

4. There is no λ_i with a large imaginary part, for which the condition $Re\lambda_i \ll 0$ is not satisfied.

The stiffness of the system for nonlinear problems described in terms of the eigenvalues of the Jacobi matrix along the curve of the exact solution. The stiffness of the nonlinear problem can be completely described in terms of stiffness for a linear problem with variable coefficients

$$\dot{\Delta}(t) = M(t)\Delta(t), t \geq t_*, \Delta(t_*) = \bar{y}(t_*) - y(t_*). \qquad (2)$$

The mathematical model of a typical bank can be rewrited in the following notation [6]:

$$
\begin{aligned}
\dot{x}_1 = -x_1\left(\frac{u_1}{\tau}+\frac{u_2}{\Delta}\right)+x_2\left(\frac{r(t)}{q(t)}+\frac{1}{\theta(t)}+\frac{1-u_2}{\Delta}\right)-& \\
-x_3\left(r_1(t)-\frac{1}{\eta(t)}+\frac{u_3}{\Delta_1}\right)+\frac{u_3kI_0}{\Delta_1}-p(t)C(t)+\Phi(t);& \\
\dot{x}_2 = x_1\frac{u_2}{\Delta}+x_2\left(\frac{\dot{q}(t)}{q(t)}+\frac{1}{\theta(t)}-\frac{1-u_2}{\Delta}\right);& \\
\dot{x}_3 = -x_3\left(\frac{1}{\eta(t)}+\frac{u_3}{\Delta}+\frac{u_3kI_0}{\Delta_1}\right), t\in[0,T].& \qquad (3)
\end{aligned}
$$

For the numerical solution of the problem of eigenvalues [7] distributed computing [8] and GRID-technologies are used.

The authors were supported by the Russian Foundation of Basic Research (project no. 15-07-08952).

REFERENCES

1. *Shalashilin V.I., Kuznetsov E.B.* Metod prodolzheniya resheniya po parametru i nailuchshaya parametrizatsiya, M.: URSS. 1999.
2. *Rakitskiy Yu.V., Ustinov S.M., Chernorutskiy I.G.* Chislennyie metodyi resheniya zhestkih sistem, M.: Nauka, 1979.
3. *Lebedev V.I.* Funktsionalnyiy analiz i vyichislitelnaya matematika. M.: Fizmatlit. 2000.
4. *Hairer E., Wanner G.* Solving ordinary differential equations II: Stiff and differential-algebraic problems (second ed.), Berlin: Springer-Verlag, 1996.
5. *Fedorenko R.P.* Vvedenie v vyichislitelnuyu fiziku. M.: MFTI, 1994.
6. *Romanyuk D. V.* Model formirovaniya kreditno-depozitnoy politiki banka, Preprint WP/97/027: M.. TsEMI RAN. 1997.
7. *Dikusar V.V., Koshka M., Figura A.* Kachestvennyie i kolichestvennyie metodyi v zadachah optimalnogo upravleniya s fazovyimi i smeshannyimi ogranicheniyami. M.: MFTI, 2001.
8. *Dolmatova A.I., Olenev N.N.* Dynamics of social stratification modeling: parallel algorithms and programs. M.:Dorodnicyn Computing Centre, 2014.