International Conference
"Mathematical Modeling of Ecological Systems"
September, 9 - 12, 2003
Almaty

AN INTERACTION MODEL FOR LIVESTOCK FARMING AND STEPPE ECOSYSTEM [1]

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This work builds a macro model for livestock farming based on aggregation of initial micro description for rational control of total number of vintage livestock (age cohorts). Solution of an optimal problem on cohort total number of livestock gives the age limit for the operation of animals, the volume of the credits for the farm, the speed of slaughter, and the conditions of the farm purchase of young animals. The integral equation for the equilibrium price of young animals is obtained. A steppe ecosystem model based on a carbon cycle is constructed. The model takes into account some external influences. It is taken into account that the maintenance of animals damages the nature.

Numerical experiments on the obtained ecological- economic model with the rational expectations is solved with the help of the highly productive calculations on the super-computer MVS 1000M. On the basis of base scenario the developments of processes in several alternatives are built and calculated. One of scenarios demonstrates a regime in which the area of natural pastures is reduced by the virtue of external human activity.

1 A Model of Steppe Ecosystem

At each moment of time t block "Ecosystem" of model describes a carbon revolution between the following components of steppe ecosystem: a stock of carbon in green phytomas of grassy vegetation $G(t)$, a stock of carbon in root system of vegetation $W(t)$, a stock of carbon in litter $V(t)$, a stock of carbon in humus $U(t)$. At each moment of time t external influences on this ecosystem are set by three economic parameters: a complex parameter of environment degradation $P(t)$; speed of consumption of carbon from grassy vegetation by grazed animals $F(t)$ and speed of restoration of carbon of a grassy cover $S(t)$.

$$\frac{dG}{dt} = Q^G(t) - Q^W(t) - Q^V(t) + S(t) - F(t). \qquad (1)$$

[1]The work is in part supported by the Russian Foundation of Basic Research (Grant 01-01-00114 and Grant 03-06-85600) and the Grant of the President of the Russian Federation for State Support of Leading Scientific Schools (no. SS 1843.2003.1).

The stock of carbon in biomass of root system W decreases by decomposition of roots biomass with speed Q^K:

$$\frac{dW}{dt} = Q^W(t) - Q^K(t). \qquad (2)$$

Besides the vegetative litter-fall Q^V and biomass decomposition Q^K additional carbon in litter V is formed also by animal excrement Q^F. Litter carbon passes in humus by its decomposition Q^U, passes in an atmosphere Q^H by breath and by airing R^V:

$$\frac{dV}{dt} = Q^V(t) + Q^K(t) + Q^F(t) - Q^U(t) - Q^H(t) - R^V(t). \qquad (3)$$

As a result of litter decomposition the stock of carbon in humus U is replenished with speed Q^U. This stock decreases in result of airing R^U and breath (allocation in an atmosphere) Q^A:

$$\frac{dU}{dt} = Q^U(t) - Q^A(t) - R^U(t). \qquad (4)$$

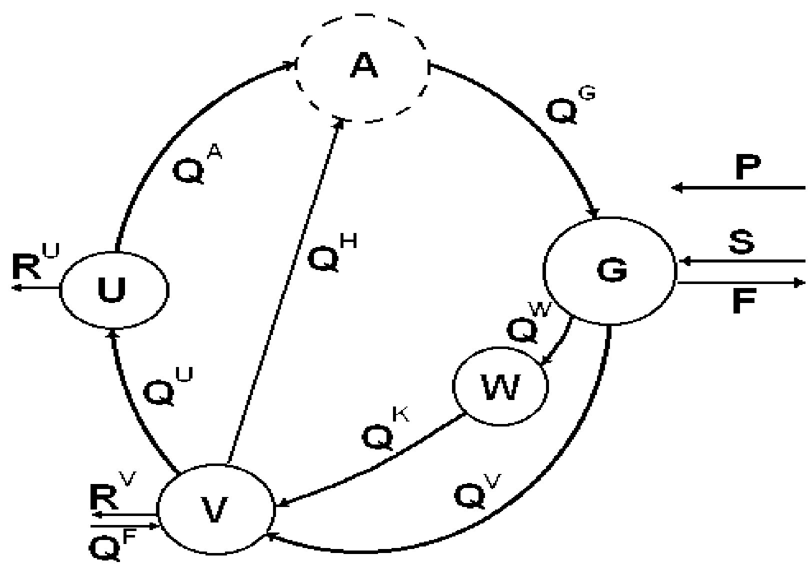

Fig.1 Carbon circulation scheme in the steppe ecosystem model ($G = 28\%$, $W = 24\%$, $V = 3\%$, $U = 45\%$)

It is supposed, that in absence of external influences the ecosystem stays in some stationary condition. Flows and stocks of carbon appropriate to this stationary condition are defined on actual flows by ratio to this stationary condition.

2 Model of Livestock Farming Sector

Let us consider the aggregated branch of livestock farming sector, which has the only one breed in it. Suppose that the production processes use homogeneous labor and the only natural resource — steppes' grassy vegetation. In [1] under this conditions the description of productions in branch of animal industries is received.

It is supposed that the households sell all young animals $n_0(t)$ on the price $q(t)$ and by desire it can always purchases on them on the same price. Then problem of economically rational behaviour of households breaks up on problems of management by separate cohorts of animals of one age. The cohorts sell a unique homogeneous product on current price $p(t)$, employ a labor under the uniform wages $s(t)$, pay the uniform per animal head tax $s_0(t)$ for each animal, pay the rent for use of a natural resource (pastures) on a flat price $g(t)$.

Cohort's total number $n(t, \tau)$ at the moment of time t is decreased in time owing to natural death rate, casual death under adverse external conditions, and also due to slaughter.

$$\frac{\partial n(t,\tau)}{\partial t} = -(d(t, t-\tau) + \Lambda)n(t,\tau) - w(t,\tau). \tag{5}$$

Traditional livestock farming is rather risky sector. Therefore in calculation of number except for expected $d(t, t-\tau, t)n(t, \tau)$ and controlled $w(t, \tau)$ reductions of a cohort, we should take into account also casual reduction with rate Λ, as its opportunity of one moment deaths entirely.

Number of newborn young animals in the moment t

$$n_0(t) = n(t,t) = \int_{-\infty}^{t} \beta(t, t-\tau)n(t,\tau)d\tau, \tag{6}$$

where $\beta(t, t-\tau) > 0$ is a fertility of animals from a cohort τ. For description of dynamics it is necessary to set also an initial number of animals in cohort $n(t_0, \tau), \tau < t_0$.

The maintenance of animals of total number $N(t)$ damages a nature $E(t)$ and demands a labor $H^D(t)$ and a raw material $B(t)$.

The production output $Y(t)$ is

$$Y(t) = \int_{-\infty}^{t} \left(k(t, t-\tau)n(t,\tau) + m(t, t-\tau)w(t,\tau)\right) d\tau, \tag{7}$$

where $k(t, t-\tau) \geq 0$ is output from one alive animal of age $t-\tau$ per unit of time at the moment t, and $m(t, t-\tau) \geq 0$ is output of production from one animal of age $t-\tau$ at the moment t. Dependence of functions $m(t, a)$ and $k(t, a)$ on a given t from age a are received on the basis of the data from [3].

For a purchase newborn animals the household can take the credit. We suppose that according to the rules of the demand by credit line all the accounts of household go through a credit balance $l(t,\tau)$. Then with the account of tax payments s_0, charge of interest rate $r(t)$ and incomes from sale of young animals

$$\frac{\partial l(t,\tau)}{\partial t} = -\pi(t,\tau)n(t,\tau) - \kappa(t,\tau)w(t,\tau) + z(t,\tau) + rl(t,\tau), \quad l(t,\tau) \geq 0, \tag{8}$$

where $\kappa(t,\tau) = p(t)m(t,t-\tau)$, $\pi(t,\tau) = p(t)k(t,t-\tau) - s(t)\lambda(t,t-\tau) - g(t)a(t,t-\tau) - s_0 + q(t)\beta(t,t-\tau)$, and $z(t,\tau)$ is pure income of cohort τ.

The credit limit is determined by a monetary estimation of each herd. Let's assume that liqudating cost $p(t)m(t,t-\tau)n(t,\tau)$ serves as such estimation. If the herd perishes the debt does not come back to bank. Therefore bank will not give the credit for all this size and it will require that a part of herd should be created on household own means.

$$l(t,\tau) \leq \sigma n(t,\tau)\kappa(t,\tau), \quad 0 \leq \sigma \leq 1. \tag{9}$$

Let's consider that the owners save up a part of the income $z(t,\tau)$ putting it in bank system. The deposits of owners are changed by the equation

$$\frac{\partial q(t,\tau)}{\partial t} = (1 - \eta(\rho(t)))z(t,\tau) + \rho(t)q(t,\tau), \quad q(\tau,\tau) = 0, \tag{10}$$

where $\rho(t) > 0$ is a deposit percent, and $\eta(\rho)$ is a function of propensity of the owners to consumption. Cumulative profit of all cohort is

$$Z(t) = \int_{-\infty}^{t} z(t,\tau)d\tau. \tag{11}$$

Economically rational decision of household on a purchase of young animals at the moment t in amount of $n(t,t)$ and on age rational slaughter of animals at this moment is determined in [1]. For the description of evolution of economy it is important how the households realize the plans. The realizable of the plans depends on the price forecasts. It seems that the modeling gets in a trap of paradox: the model is necessary to help household to predict the prices, and for construction of model it is necessary to know the price forecast. The paradox is solved by application of the rational expectations hypothesis [4]: on required time horizon the households predict the prices correctly.

It is shown in [1] that the household irrespective of the expected prices takes the greatest possible credit if $r \leq \rho + \Lambda$, and it refuses to use the credit otherwise. They slaughter wholly a cohort of animals born at the moment τ in the age θ when the next inequality is infringed at the first time

$$\frac{\pi(t,\theta)}{\kappa(t,\theta)} - d(t,\theta) - \Lambda - \rho(t) + (\rho(t) + \Lambda - r(t))_+\sigma(t) + \\ + \frac{1}{m(t,\theta)}\frac{dm(t,\theta)}{dt} + \frac{1}{p(t)}\frac{dp}{dt} > 0. \tag{12}$$

In [1] the following integrated equation for the equilibrium price $q(t)$ is received

$$q(t) = p(t)m(0,t) + \\ + \int_t^{\infty} [(-[\rho + \Lambda - r]_+ \sigma + \Lambda + d(x, x-t) + \rho) p(x) m(x, x-t) - \\ - p(x) k(x, x-t) + s(x)\lambda(x, x-t) + s_0 - q(x)\beta(x, x-t) - \\ - \partial(p(x)m(x, x-t))/\partial x]_- \exp(\int_x^t (d(\xi, \xi - t) + \Lambda + \rho) d\xi) dx. \qquad (13)$$

It is visible that this equation contains unknown forward-looking values. The solutions of such equations have a characteristic complex oscillatory behaviour that reflects specifity of livestock farming sector.

3 Closing the model

It is supposed that price on goods market is changed with stocks. The complex parameter of environment degradation $P(t)$ is determined by

$$\frac{dP}{dt} = \big(E(t) - d^P \Phi^T(t)/p(t)\big)/A - \alpha^P P(t), \quad (P(t) \geq 0), \qquad (14)$$

where A is a total square of pastures, $\alpha^P > 0$ is a tempo of natural cleaning, $\Phi^T(t)$ is a fund for cleaning, $d^P > 0$ is an effectiveness of using the fund.

The total population attached to the sector $H(t)$ grows with fix rate $\gamma(t)$, a labor supply $H^S(t)$ is determined by active part of population $H^A(t)$ with coefficient χ depended on a consumption level $\omega^R(t)$. The wage $s(t)$ is quickly grows up when the labor demand H^D is greater the labor supply H^S, otherwise it slowly changed with inflation.

The monetary system is set by model of currency board, where gold-currency reserve $R(t)$ is changed by balance of payments

$$\frac{dR}{dt} = (p(t)/w)X(t) - I(t), \qquad (15)$$

where w is a fix exchange rate of national currency. Export flow $X(t)$ is recounted by ratio $p(t)/w$ in a foreign hard currency in which import flow $I(t)$ is measured. The bank assets consist of gold-currency reserve $R(t)$ and total cohort debts $L(t)$, and liabilities consist of deposits $D(t)$:

$$wR(t) + L(t) = D(t), \qquad (16)$$

Total volume of crediting is determined by

$$C^S(t) = \frac{(1-\xi)}{\xi}\big(p(t)X(t) - wI(t)\big) + \Lambda L(t) - J(t), \qquad (17)$$

where ξ is reserve norm ($R \geq \xi D$), $J(t) = \int_{n(t,\tau)\geq 0} \frac{\partial}{\partial t} l(t,\tau) d\tau$. The deposit procent $\rho(t)$ is determined by

$$\rho(t) = \frac{(p(t)X(t) - wI(t))/\xi - (1-\eta)Z(t))}{D(t) + (1-\eta)L(t)}, \tag{18}$$

where η is consumption norm of owners.

The part of crediting $\sigma(t)$ is determined by equality

$$\sigma(t) = \frac{C^S(t)}{p(t)O(t)}. \tag{19}$$

where $O(t)$ is a total cost of cohorts' dismantling.

4 Numerical experiments with model

Fig.2-3 show the dynamics of some macroindexes of the ecologo- economic model in calibrated (base) version.

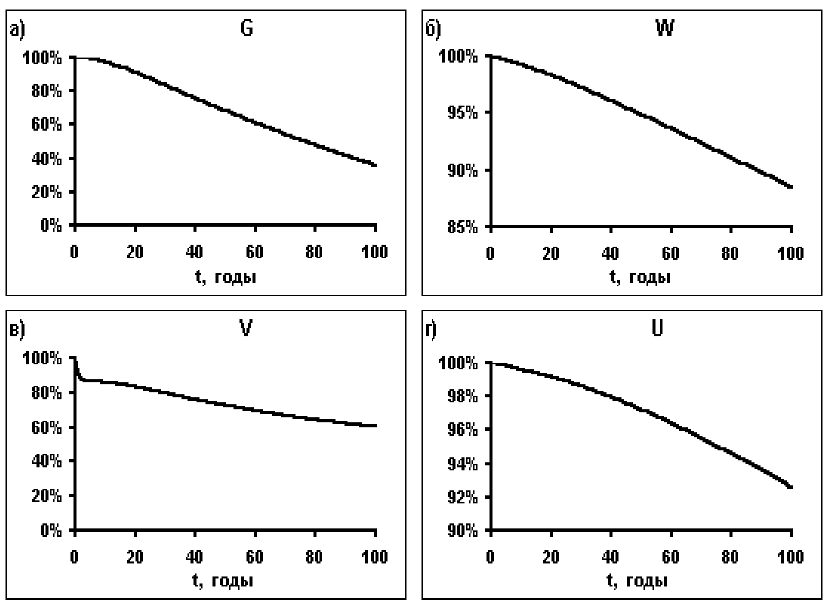

Fig.2 Main indexes of steppe ecosystem

Green phytomas of grassy vegetation $G(t)$ falls by almost linear trend to 36% of stationary level. Root system of vegetation $W(t)$ falls to 89%. Litter $V(t)$ falls to 60%, humus $U(t)$ falls to 93%.

The total number of flock N (Fig.3 a)) grows up in 2.5 times practically by linear trend, the toatal number of newborns M (Fig.3 б)) grows up on the same value. Influence of economy on ecosystem is increased: complex parameter of environment degradation P (Fig.3 в)) grows up from 0 points to danger level of

3.4 points for 100 years. It gives seriously contribution on state of ecosystem (see Fig.2). consumption of carbon from grassy vegetation by grazed animals F (Fig.3 г)) grows up in 2.3 times over stationary level.

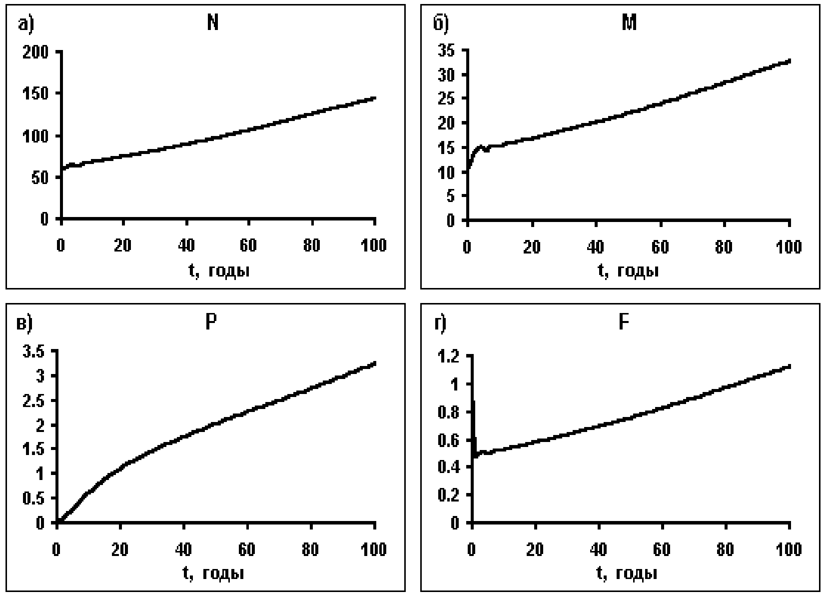

Рис.3 Some macroindexes of livestock farming sector

As a scenario let's consider linear lowering of the total square of pastures on 12% for 100 years. the area of natural pastures is reduced by the virtue of external human activity (tourism and industries activities).

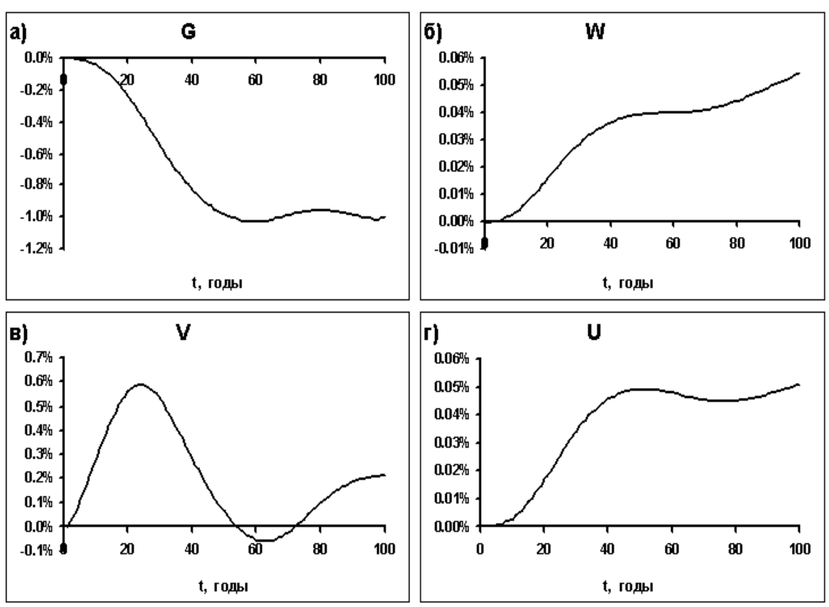

Fig.4 Comparison of main indexes for steppe ecosystem

The dynamics of variations for scenario and base indexes is shown on Fig.4-5. The calculation of variation is determined by

$$Var(X(t)) = \left(\frac{X^C(t)}{X^{Б}(t)} - 1 \right) \cdot 100\%,$$

where X^C is a value of an index for scenario version and $X^{Б}$ is a value of the proper index for base version.

On each unit of pasture square Green phytomas of grassy vegetation $G(t)$ (Fig.4 a)) falls on 1.0% in comparison of scenario version with base one. At the same time litter biomass (V, Fig.4 в)) in scenario version is fluctuated near level of base version. Biomass of root system of vegetation (W, Fig.4 б)) and humus $U(t)$ practically don't change.

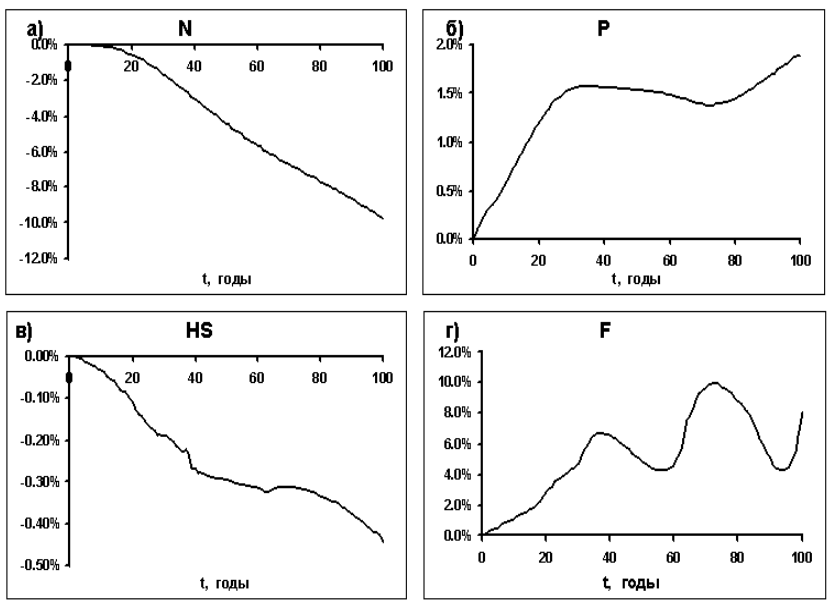

Fig.5 Comparison of some macroindexes of livestock farming

Influence on macroindexes of livestock farming is more greater. Total number of livestock fall down on 10% (N, Fig.5 a)). The complex parameter of environment degradation (P, рис.5 б)) grows up to 2%. Consumption of carbon from grassy vegetation by grazed animals (F, Fig.5 г)) grows up on 10%. Labor supply falls down on 0.4%.

5 Conclusion

On the basis of the given mathematical model the numerical simulation model is constructed. It is calibrated on the Mongolian statistical data. This model is capable to give qualitatively correct quantitative valuations to main tendencies in development of transition economy, environment and their interaction. The given numerical simulation model is an irreplaceable tool for construction of the alternative scripts of future development and it also enables to evaluate consequences of these scripts realization.

Traditional pasturable livestock farming does not require large labor costs, however it essentially depends from external meteorological conditions and it also renders huge influence on grassy steppe ecosytem, causing its degradation in case of excessive loading. Investigation of dynamics of influence of economic growth on an environment shows that considering of reorganization of economic structures, i.e. transition from traditional pasturing livestock farming to more labor-consuming and capital-intensive household management will result in improvement of an environment conditions.

Authors count the pleasant debt to express deep gratitude to Alexander Petrov for constant attention to the work.

References

[1] *Demberel S., Olenev N.N., Pospelov I.G.* To a mathematical model of interaction of economic and ecological processes. // Mathematical modeling. 2003. Vol.15. No.4. P. 107-121. In Russian.

[2] *Demberel S., Olenev N.N., Pospelov I.G.* Interaction of Livestock Farming and Steppe Ecosystem . Moscow: Dorodnicyn Computing Center, 2003. 40 p. In Russian.

[3] *Nyamaa* Biological bases and ways of increase of production of mutton. Doctor's (Sc) Thesis. Ulanbaatar. 1994. In Mongolian.

[4] *Muth J.F.* Rational Expectations and the Theory of Price Movements. // Econometrica, Vol. 29, No. 3. (Jul., 1961), pp. 315-335.