# Short course
# Numerical Grid Generation: State-of-the-art & State-of-the-practice
## June 8-9, 2008

International conference "Numerical geometry, grid generation and scientific computing" and International workshop "Voronoi-2008" honoring 140th anniversary of G.F. Voronoi, June 10-13, 2008, Moscow, Russia.
http://www.ccas.ru/gridgen/numgrid2008

## Dorodnicyn Computing Center of Russian academy of sciences
## 119333 Vavilov str. 40, Moscow, Russia

## Lecturers:
**Prof. N.P. Weatherill, University of Swansea, UK,**
**Prof. B.K. Soni, University of Alabama at Birmingem, USA,**
**Dr. V.A. Garanzha, A.A. Dorodnicyn Computing Center RAS**

**AGENDA**

Sunday, June 8, 2008

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|---|---|
| 9:00 – 9.15 | Introductions. |
| 9.15 – 10.15 | Overview of Mesh Generation & Processes and Applications. B.K. Soni |
| 10.15 – 11:00 | Computer Aided Geometry Design. B.K. Soni |
| 11:00 – 11.15 | Break |
| 11:15 – 12:00 | Surface Mesh Generation. N.P. Weatherill |
| 12:00 – 12:30 | Discrete curvatures. V.A. Garanzha |
| 12:30 – 2:00 | Lunch |
| 2:00 – 3:00 | Unstructured Meshing. N.P. Weatherill |
| 3:00 – 4:00 | Automatic Hex Dominant Meshing. B.K. Soni |
| 4:00 – 4:30 | Break |
| 4:30 – 5:30 | Parallel Meshing. N.P. Weatherill |

Social Dinner

Monday, June 9, 2008

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| 9:00 – 9:30 | Mesh adaptation: general concepts. N.P. Weatherill |
| 9:30 – 10:15 | Mesh Adaptation – Structured Meshing. B.K. Soni |
| 10:15 – 11:00 | Mesh Adaptation – Unstructured Meshing. N.P. Weatherill |
| 11:00 – 11:30 | Break |
| 11:30 – 12:30 | Variational methods for optimal meshing. V.A. Garanzha |
| 12:30 – 2:00 | Lunch |
| 2:00 – 2:45 | Unstructured Meshing – Future Directions . N.P. Weatherill |
| 2:45 – 3:30 | Meshing Software, State of the Practice & Future Directions. B.K. Soni |
| 3:30 – 4:00 | Break |
| 4:00 – 5:30 | Q&As & Discussions (All Instructors, Moderator: V.A.Garanzha) |

## Abstracts of lectures

**Overview of Mesh Generation & Applications**

Mesh generation continues to remain pacing crosscutting enabling technology that limit the efficiency of computational field simulations (CFS) for complex configurations. Rapid turn-around time, reliable and accurate simulations and affordability/economy are the key ingredients needed for CFS applicability to routine design and acquisition process.

General overview of mesh generation strategies and processes with wide spectrum of real-world applications will be presented. Mesh generation considerations involving structured/unstructured grids with point and cell distributions will be discussed with truncation errors, stretching functions, as well as mathematics/numerical methods employed in the overall discretization as well preprocessing approach. A step-by-step process of mesh generation will be described introducing the topics to be covered in this short course.

**Computer Aided Geometry Design**

Geometry modelling and manipulation consumes 70-90% of human-time associated with complex mesh generation application. Computer Aided Geometry Design (CAGD) principles and algorithms play a very important role in geometry treatment essential for surface meshing and hence volume mesh generation.

Basic principles of CAGD along with the development of Bezier, Hermite, B-Spline and NURBS (Non Uniform Rational B-Spline) curves and surfaces will be presented. The properties and characteristics of CAGD methods will be discussed in light of mesh generation. Geometry specifications IGES and STEP will be described with their role in industry.

The application of CAGD techniques in surface/volume mesh generation will be presented. These applications will be illustrated using real-world complex configurations in wide spectrum of disciplines.

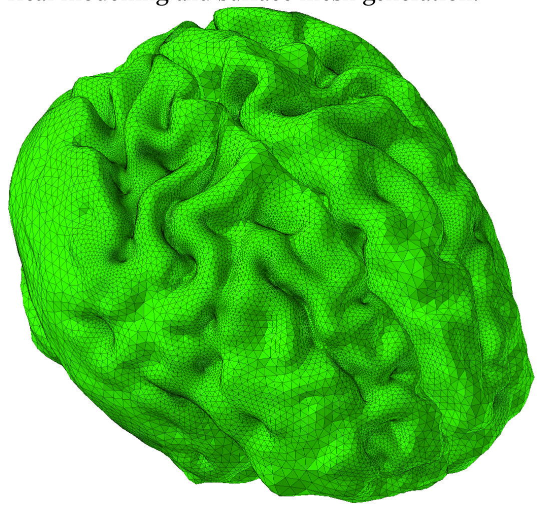
**Surface Mesh Generation**

The generation of high quality meshes on complex geometrical surfaces is a challenge. However, it is a fundamental step in the construction of volume meshes, since the quality of the surface mesh influences the quality, and in many cases the robustness, of a volume mesh.

The lecture will begin with some background information on the mathematical representation of surfaces. Further discussion will cover how geometries of practical objects are defined, links to CAD systems and the extraction of appropriate data, such as geometrical topology. The general issue of the generation of meshes on surfaces will then be discussed and how the classical parameterisation of a surface can be used, coupled with two-dimensional mesh generators (structured and unstructured), to produce surface meshes that conform to the underlying geometry.

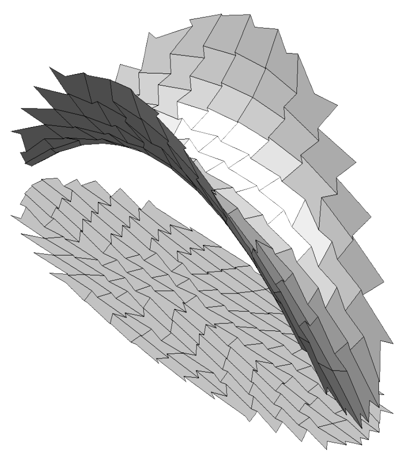
Following discussion onthe basic approaches, the lecture will then move to cover more advanced topics including the combining of surfaces to create super-surfaces and the meshing based upon triangulations of a surface and the basic STL format.

The lecture will conclude with some applications, including aerospace and bio-medical and an overview of present and future challenges in geometrical modelling and surface mesh generation.



**Discrete curvatures**

Approximation of surfaces by polyhedra is challenging problem of modern discrete differential geometry. Computation of curvature of polyhedral surfaces is important problem in animation, pattern recognition, geometric modeling, reverse engineering, structural mechanics, computational biology, as well as in interface-related problems in physics, chemistry and biology.



Intrinsic curvature computation is based on length measured along surface and integral Gauss-Bonnet relations, while extrinsic curvature is based on the concept of spherical Gauss map. One can compute discrete extrinsic curvatures using relation between mean curvature and surface Beltrami operator, or relation between variation of surface area and swept volume. Duality principle allows to compute discrete curvature tensor and discrete approximation to integral curvature measures of the surface and to discern geometric noise from surface features.

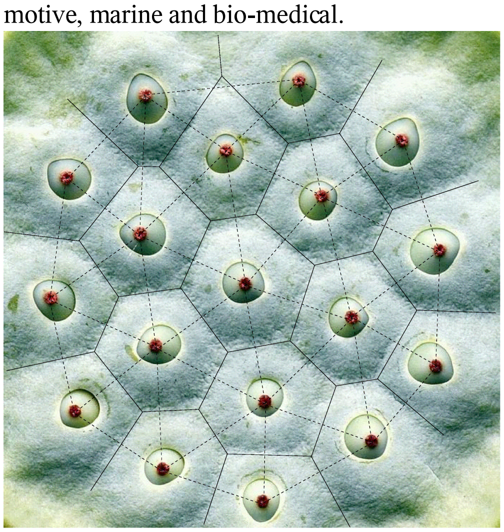
### Unstructured Mesh Generation

Unstructured mesh generation is based on a tessellation of a domain, where the elements in the tessellation are not constrained by any particular underlying structure or implied mapping. Hence, nodal connectivities are used to define the elements. This flexibility is a powerful attribute of unstructured meshes that has resulted in (semi-) automatic meshing of arbitrarily complex domains.

This lecture will introduce the basics of unstructured mesh generation before focusing on details of two methods that can be used to generate unstructured meshes of triangles and tetrahedra.

The advancing front method and the Delaunay/Voronoi methods are powerful techniques for the generation of unstructured meshes. Details will be provided on both these approaches, including algorithm implementation details that ensure computational efficiency and speed.

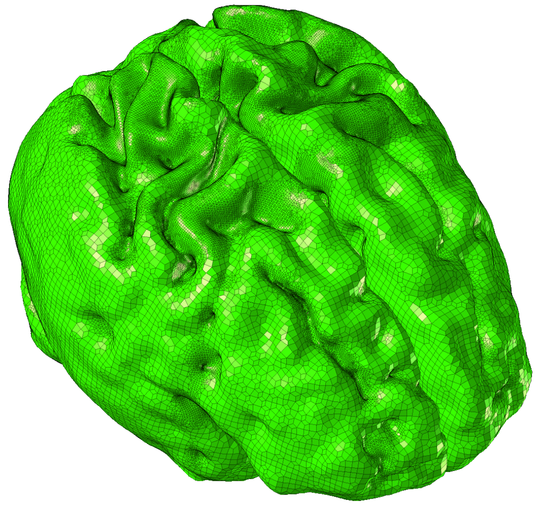
These methods will be illustrated on real-world geometries and examples will be given from a range of subject areas, including aerospace, automotive, marine and bio-medical.



### Automatic Hex Dominant Meshing

All structured or hex dominant meshes are very attractive in computational field simulations for better accuracy and efficiency of the numerical schemes. However, generation of all-hex mesh, especially on a complex configuration, is a challenging task.

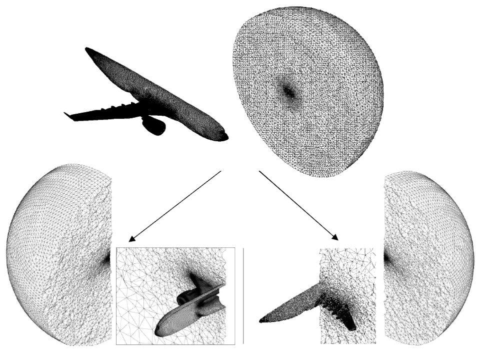
The lecture will discuss meshing techniques and approaches with focus on hex-dominant grids. Emphasis will be placed on automatic meshing schemes. Advancing layer type methods, octree-based schemes, weaving/knitting techniques as well as algebraic and PDE based technologies will be presented with complex applications involving several disciplines. A scripting/template based approach of automation applicable to design problems will be described.



### Parallel Mesh Generation

With the advent of parallel computers and the increase in size and complexity of computation that this affords, there has been a major effort to construct meshes using more than one processor, that is, so-called parallel mesh generation. The state-of-the-art in computation for some applications, for example, electromagnetic scatter, requires meshes of 100’s of millions of elements and the parallelisation of the process of mesh generation is therefore very important.

The lecture will discuss the motivation for parallel mesh generation and then discuss the different approaches that can be taken. Mesh generation algorithms can be reconstructed in a parallel framework and/or the parallelisation can be achieved by dividing the domain into sub-domains. Both these approaches will be discussed for structured and unstructured meshes. Details will be given for parallel surface and volume mesh generation.

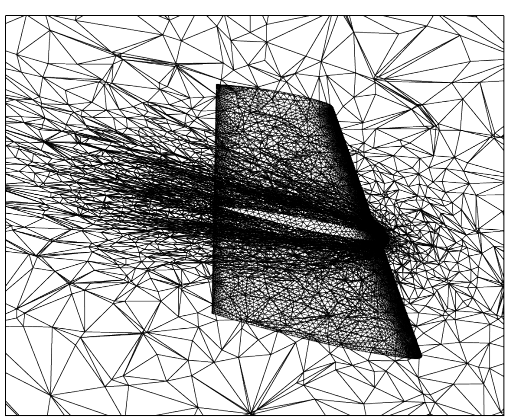


The lecture will consider real-world examples, where unstructured meshes of one billion elements have been generated in parallel.

## Mesh Adaptation: General Concepts

Following the construction of a mesh, the solution algorithm is then employed to model the phenomena in question (e.g. heat transfer, fluid flow etc). The solution will be an approximation to the exact solution and the issue of how the mesh can be changed to minimize these errors is at the heart of the process called mesh adaptation.

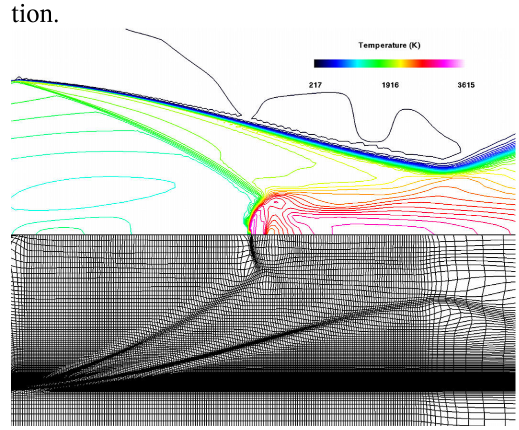
This is a highly challenging area of research and the answer to this question is at this stage far from clear. The first issue is how can we assess the errors, when we do not know the exact solution? (This is generally the case, apart from simple test case problems.) This issue is complicated by the complexity of the solution equations, which are often highly non-linear. Even if estimates could be made, the next issue is how is a mesh changed to reduce the errors?



The lecture will cover some of these fundamentally important issues, with a discussion on error estimates and error analysis, followed by a general discussion on mesh adaptation procedures and strategies.

## Mesh Adaptation: Structured Meshing

The accuracy of numerical simulation of a physical field problem depends not only on the formal order of approximation but also on the distribution of mesh points in the computational domain. The quality of the mesh based on the geometric characteristics influenced by the numerical scheme under consideration and solution characteristics representative of the field properties being simulated is extremely important in improving the accuracy and convergence rate of the simulation.




The lecture will cover grid quality evaluations and adaptation procedures for structured meshes. Truncation errors associated with finite difference/finite volume schemes will be analyzed to describe importance of grid stretching, aspect ratio, smoothness and near orthogonality of grids. Strategies employed in dynamically adaptive grids coupled with the PDEs of the physical problem will be described. Equidistribution principle applied to algebraic and elliptic equations for mesh adaptation based on re-distribution will be described.

Software and algorithms for mesh adaptation will be presented with computational examples of wide interest.

## Mesh Adaptation – Unstructured Meshing

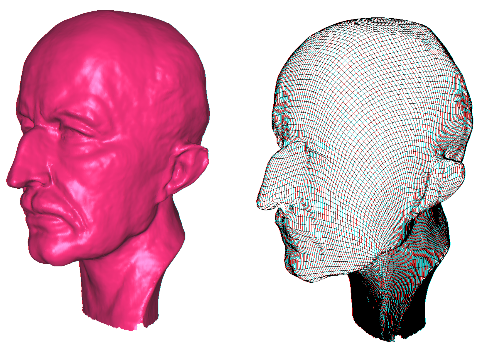
Unstructured meshes, given the lack of underlying structure in the element connectivities, provide a flexible and practical framework for mesh adaptation techniques. Nodes in the mesh can be added or removed to help reduce errors without changing the fundamental form of the mesh and, if required, regions of a mesh can be re-generated.

The lecture will discuss in detail the approaches of mesh enrichment, mesh de-refinement, mesh node movement and remeshing. In addition, other approaches where unstructured meshes can be used to adapt to physical features, such as wakes, vortices, shock waves will be described.

These techniques will be illustrated on real-world problems in both two and three dimensions.

**Variational methods for optimal meshing**

Variational methods are
Modern well-posed variational mesh generation methods are based on construction of one-to-one mappings between surfaces/manifolds. They provide unified robust algorithms to optimize meshes consisting of different types of elements: tetrahedra, prisms, hexahedra, polyhedral cells, B-spline elements, etc., in terms of size, shape and alignment. Robust numerical computations in the presence of nonregular adaptation metric. Efficient minimization methods for discrete functionals are considered. Coupling of mesh optimization via node movement with topological mesh optimization is discussed.



**Unstructured Meshing – Future Directions**

Is the problem of unstructured mesh generation solved? This will be the focus of the initial discussions and it will be clear that there are still many challenging problems to be faced.

Areas for future research will be considered and the lecture will include some latest research that attempts to improve on present techniques, whilst new approaches will be discussed that could lead to significant advances in the area.

The lecture will be illustrated with challenging problems that cannot yet be solved.

**Meshing Software, State of the Practice & Future Directions**

With the advent and rapid development of high performance computing and communication (HPCC) technology computational fluid dynamics (CFD) and computational structural dynamics (CSD) have emerged as essential tools for engineering analysis and design. Last three decades have seen considerable progress in the development of tools and technologies for addressing CFD and CSD applications. However, **accuracy, confidence, thru-put and cost effectiveness in performing CFD and CSD simulations** remain the critical barriers associated with complex applications.

Historical perspective with current state-of-the-art and state-of-the-practice will be presented by summarizing strategies and associated simulation schemes, mesh generation algorithms and commercial and public-domain software, data standard efforts, geometry treatment approaches, mesh adaptation, moving/deforming mesh generation, automation of meshing timely enough for design optimization, and computer science tools. Cartesian, structured, unstructured, hybrid/generalized meshing strategies along with mesh-less methods will be discussed in conjunction with wide spectrum of application disciplines will be included in these perspectives.

Current critical barriers will be discussed with future directions with respect to continuously growing application complexities and information technology.