

# Metric tensors for generation of optimal meshes

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We consider optimal meshes that minimize the error of the piecewise linear interpolation over all simplicial meshes with a fixed number of cells  $N_T$ . We analyze both the interpolation error  $\|u - \mathcal{I}_{\Omega_h} u\|_{L_p(\Omega)}$  and the gradient of interpolation error  $\|\nabla(u - \mathcal{I}_{\Omega_h} u)\|_{L_p(\Omega)}$  and present theoretical results on asymptotic dependences of the errors on  $N_T$ . In practice, the conventional adaptive procedures produce meshes close to optimal. Such meshes are called quasi-optimal. They give slightly higher errors but the same asymptotic rate of error reduction. Quasi-optimal meshes are uniform or quasi-uniform in an appropriate continuous tensor metric [1,2]. Generation of triangular and tetrahedral quasi-optimal meshes is implemented in public libraries

<http://sourceforge.net/projects/ani2d>

<http://sourceforge.net/projects/ani3d>

Metric recovery is the cornerstone of the mesh adaptation. It is usually based on either a discrete Hessian recovery or a posteriori error estimators. We discuss difficulties related to both approaches and propose possible solutions. Three methods of the discrete Hessian recovery are considered and compared. It is known that the accuracy of the Hessian recovery is very low although the method exhibits surprisingly good behavior in practice. A posteriori error estimators may provide a reliable alternative for metric recovery [3]. We explain why edge-based estimators are preferable to element-based estimators and show that quasi-optimal meshes may be generated using the edge-based estimators.

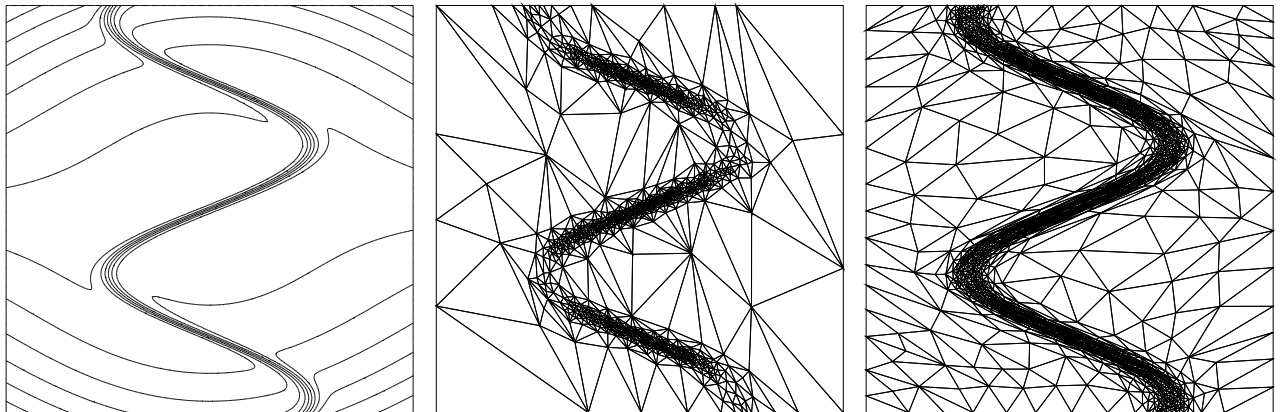


Figure 1: Isolines of an anisotropic function  $u$  (left), quasi-optimal meshes minimizing  $\|\nabla(u - \mathcal{I}_{\Omega_h} u)\|_{L_p(\Omega)}$  (center) and  $\|u - \mathcal{I}_{\Omega_h} u\|_{L_\infty(\Omega)}$  (right);  $N_T = 2500$ .

1. Y.Vassilevski, A.Agouzal. An unified asymptotical analysis of interpolation errors for optimal meshes. // Doklady Mathematics, 72, 879-882, 2005.
2. W.Huang. Metric tensors for anisotropic mesh generation. // J. Comp.Phys., 204, 633-665, 2005.
3. A.Agouzal, K.Lipnikov, Y.Vassilevski. Generation of quasi-optimal meshes based on a posteriori error estimates. // Proceedings of 16th International Meshing Roundtable, 2007.